Minimum Wages, Inequality, and the Informal Sector*

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Abstract

How do minimum wages affect earnings inequality in countries with large informal sectors? I provide reduced-form evidence that the 2000s minimum wage hike in Brazil raised overall inequality by increasing inequality inside the informal sector. I develop a model where heterogeneous firms select into informality to investigate when and how raising the minimum wage can increase inequality. I calibrate the model to Brazil and find that, by generating substantial informality, the increase in the minimum wage raised overall inequality by 6.4%. These results suggest that movements into and out of the informal sector modulate the effects of formal labor legislation.

Earnings inequality fell substantially in Latin America throughout the 2000s.[1] Minimum wage policies were found to be among the most important drivers of these patterns.[2] However, most studies do not account for the fact that firms and workers

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[1] See Lustig et al. (2013) for Latin America; Firpo and Portela (2019) and Alvarez et al. (2018) for the case of Brazil.

[2] See Engbom and Moser (2021) and Haanwinckel (2020).

can avoid labor legislation by operating informally — a ubiquitous feature in most low income and developing countries.[3] Hence, when evaluating the consequences of the minimum wage for inequality, it is important to incorporate this margin of adjustment. What are the effects of the minimum wage on inequality when the informal labor market is taken into account?

This paper proposes answers to this question in three steps. First, using Brazilian survey data on both formal and informal labor markets, I show that inequality in the informal sector did not fall alongside the rapid expansion of the minimum wage. Moreover, I provide reduced-form evidence that the minimum wage increased overall inequality in states most exposed to it because of strong inequality-increasing effects on the informal sector. Second, I develop a stylized model of monopsonistic competition with informality and a minimum wage to investigate under which conditions raising the minimum wage can increase overall earnings inequality and reduce worker welfare. Third, I develop a quantitative model that additionally features heterogeneous workers and skill-biased technical change, two other important drivers of informality and inequality. I calibrate the model to Brazil in the 2000s and show that, all else equal, the increase in the minimum wage is responsible for a 6.4% increase in the variance of the aggregate log earnings distribution. I also estimate that, to overcome these *unintended consequences*, government authorities should have increased their efforts to enforce formality by 85%. Moreover, I find that improvements in the skill composition of the labor force can complement minimum wage policies in reducing inequality. All in all, these findings suggest that movements into and out of the informal sector modulate the effects of formal labor legislation like the minimum wage.

In Section 1, I use Brazilian household survey data from 1996-2012 to establish stylized facts on informality, inequality and the minimum wage. First, I document that informal workers constitute 35% of the labor force, earn lower wages, and are substantially less educated than formal workers. Second, I highlight that while the variance of log earnings in the formal sector fell sharply from 0.65 to 0.33, inequality in the informal sector remained constant at 0.65. Third, I show that the minimum wage became substantially more binding in the formal sector. The share of formal workers at the minimum wage, stable around 7% until 1999, increased sharply to 16% by 2006, stabilizing at that level thereafter.

I then provide reduced-form evidence on the relationship between the minimum

[3] See Tornarolli et al. (2014) for the analysis in Latin America.

wage, inequality, and informality. To do so, I exploit state-level heterogeneity in initial exposure to the minimum wage, measured by the share of formal minimum wage workers in 1999. I find that the most exposed states experienced a 25.3 percentage points (p.p.) stronger reduction in the variance of log earnings in the formal sector (formal inequality), a 31.6 p.p. larger increase in informal inequality, and a 7.3 p.p. larger increase in the informal share compared to the least exposed states. Jointly, these led to a 20 p.p. relative increase in overall inequality in the states where the minimum wage binds the most. Moreover, I show that the effect of the minimum wage on overall inequality varies widely across states: minimum wage increases inequality in the most exposed states but decreases it in less exposed ones. These differences occur because the minimum wage has stronger effects on the informal sector in more exposed states.

Motivated by these findings, Section 2 develops a stylized model where heterogeneous firms compete for labor subject to the minimum wage and can choose to operate informally. Firms trade off minimum wage restrictions, when formal, versus revenue losses due to government inspections, when informal. In equilibrium, the most productive firms operate formally, and, within formal firms, the least productive ones bunch at the minimum wage. I derive sharp analytical results on the effect of the minimum wage on inequality. First, I decompose this effect into the formal and informal sector responses. On the one hand, a higher minimum wage compresses the earnings distribution in the formal sector. On the other hand, some formal workers lose their job and become informal. I show that the strength of the informal response depends on the elasticity of the informal sector to the minimum wage and on the differences in means and variances between the formal and informal wage distributions. Second, I show that, when the distribution of firm productivity is Pareto and informality levels are low, increasing the minimum wage increases earnings inequality. These results suggest that there can be *unintended consequences of the minimum wage*: a policy aimed at reducing inequality might end up increasing it due to strong informal margins of adjustment.

In the following two sections, I outline the quantitative model and the calibration results. Section 3 extends the stylized model and incorporates worker heterogeneity and skill-biased technical differences in production. These quantitative features were shown to be important drivers of the informal share of labor and earnings inequality,[4] and might interact with the way in which minimum wage increases

[4]See Haanwinckel and Soares (Forthcoming) and Haanwinckel (2020).

affect the economy. In Section 4, I separately calibrate the model to Brazilian data in 1996 and 2012. The calibrated framework replicates the observed distribution of wages in the aggregate economy, within each sector, as well as within each skill group.

In Section 5, I quantify the effect of the increase in the minimum wage on earnings inequality holding all other factors constant. I find that the spike in the minimum wage over the 2000s increased overall inequality by 6.4%, despite reducing formal sector wage inequality by 12.1%. This result comes from the fact that the minimum wage generated a substantial amount of informality, increasing inequality in this sector, and more than compensating for the inequality-reducing effects in the formal sector.

Lastly, I quantify the effects of the changes in formal enforcement, skill composition, and technological skill bias. First, my results suggest that the estimated increase in formal enforcement does little to prevent the minimum wage from increasing overall inequality. I estimate that an increase of at least 85% in the cost of informality is needed in order for the minimum wage to reduce inequality. Second, I find that the improvement in the skill composition reduces informality by 41%, in line with the literature on the determinants of the informal sector (Haanwinckel and Soares, Forthcoming). Informal firms are more intensive in low skill workers. Improvements in the skill composition make this factor of production more scarce, pushing up low skill wages, and increasing the associated costs of being informal. Moreover, I highlight that improvements in the quality of the labor force can complement minimum wage policies in reducing overall earnings inequality in countries with a large informal sector. Third, I show that the model-implied skill biased technical change increases the share of the informal workforce by 50% and the overall earnings inequality by 26%.

Related research. This paper contributes to three strands of literature. First, I relate to the body of work that studies the informal sector in developing economies, summarized in Ulyssea (2020). Within these papers, I contribute to the empirical work on the informal sector (Porta and Shleifer, 2008; La Porta and Shleifer, 2014; Almeida and Carneiro, 2012; Engbom et al., 2021) by providing reduced-form evidence suggesting that minimum wages can increase overall inequality when there are strong effects on the informal sector. My work is also related to the set of papers that incorporate the informal sector in models of firm heterogeneity (Ulyssea, 2010;

Leal Ordóñez, 2014; Meghir et al., 2015; Ulyssea, 2018; Dix-Carneiro et al., 2021). I complement this literature by providing a quantitative model of the minimum wage and the informal sector that delivers realistic wage distributions, and using it to quantify the effects of the minimum wage on the economy.

Second, this paper relates to the literature that studies the effects of the minimum wage on the formal sector.[5] This paper is particularly related to Engbom and Moser (2021) and Haanwinckel (2020), which study the Brazilian context using two different quantitative approaches. I make two main contributions. First, I provide reduced-form evidence that increasing the minimum wage increases overall inequality in highly exposed states relative to the states least exposed. Second, I develop a quantitative model where firms select into the informal sector to quantify the effects of the minimum wage and other mechanisms such as changes in relative supply of skills on inequality and informality.

Third, there is a related literature that studies the effects of the minimum wage in economies with a large informal sector. Jales (2018) develops a density discontinuity design to estimate, in a reduced-form way, the effects of the minimum wage on the joint distribution of employment and wages in Brazil. Jales and Yu (2020) develop a bargaining model featuring compensating differentials and self-selection to microfound the findings in Jales (2018). Derenoncourt et al. (2021) investigate the effects of the minimum wage on racial inequality and the informal sector. Using a reduced-form approach, they show that minimum wage increases are important in explaining the fall in racial earnings gap in the 2000s, but have little effects on informal labor. Haanwinckel and Soares (Forthcoming) develop a quantitative model to study the main drivers behind movements in the informal share of labor. They find that the improvement in the education composition of the Brazilian labor force was the main force behind the fall in informality, and that the minimum wage helped keeping the informal share at high levels.[6] I contribute to this literature by developing a quantitative model that delivers realistic wage distributions in the

[5] See Card and Krueger (1993), Lee (1999), Dickens and Manning (2004), Autor et al. (2016), Card et al. (2018), Harasztosi and Lindner (2019), Dustmann et al. (Forthcoming), Engbom and Moser (2021), Haanwinckel (2020), Berger et al. (2021), among others.

[6] The last section of Haanwinckel and Soares (Forthcoming) discusses the implications of their model for the effect of the minimum wage on relative wages across skills. My analysis contributes and extends theirs in two ways. First, I analyze the effects of the minimum wage not only on the relative earnings across educational groups, but relative earnings across formality status. Second, and most importantly, I show that there are significant effects of the minimum wage on inequality within the formal and informal sectors.

aggregate as well as within the formal and informal sectors. This allows me to perform counterfactual exercises and to assess the general equilibrium effects of the minimum wage on inequality within each sector as well as in the aggregate.

The rest of the paper is structured as follows. Section 1 uses Brazilian data to highlight the importance of the informal sector when evaluating the effects of the minimum wage. Section 2 develops a stylized model where the presence of informality can substantially alter the effects of the minimum wage on inequality and welfare. Section 3 extends the stylized model and introduces other mechanisms that are important in generating the observed changes in inequality and informality in Brazil. Section 4 discusses the calibration and validation. Section 5 performs the counterfactual exercises. Section 6 concludes.

1 Empirical motivation

In this section, I use Brazilian data between 1996 and 2012 to provide evidence that the informal sector modulates the effects of the minimum wage on inequality. First, I introduce the data set. Second, I present stylized facts on the informal sector, earnings inequality, and the minimum wage in the Brazilian context. I show that a large share of the workforce is informal, that earnings inequality between informal workers, differently from that in the formal sector, did not decrease over the 2000s, and that the minimum wage became substantially more binding over time. Third, I provide reduced-form evidence on the effects of the minimum wage on inequality and informality. I find that the minimum wage increases overall inequality in the states most exposed to it, due to its effects on the informal sector. Moreover, I find that the effects on the informal sector are small in less exposed states. As a consequence, the effect of the minimum wage on overall inequality varies widely across states: minimum wage increases inequality in most exposed states but decreases it in the least exposed ones.

1.1 Data

The main data set is the *Pesquisa Nacional por Amostra de Domicílios* (PNAD) over the years of 1996-2012. The PNAD is a household survey with national coverage administered by the *Instituto Brasileiro de Geografia e Estatística* (IBGE), and it is one of the primary sources of nationally representative labor market and demographics data. The PNAD data is particularly well suited for this analysis

as it contains data on both formal and informal work arrangements, which I detail below.[7] I restrict attention to individuals highly attached to the labor force (age between 18-54), and consider only one job per worker (their main job at the reference week).[8] I deflate all nominal variables by the CPI, and express them in terms of 2012. I follow the empirical literature on Brazil and consider monthly gross earnings as the main earnings measure. Importantly, the minimum wage in Brazil is de facto imposed at the monthly earnings level.[9]

In PNAD, households are asked whether they have a signed working permit (*Carteira de Trabalho Assinada*). When an employer signs its employee's working permit, that labor contract becomes subject to labor legislation such as the minimum wage, unemployment benefits, and others. Throughout this paper, a worker is informal if they do not have a signed working permit. Even though the share of self-employed workers is about as big as that of workers without a signed working permit, I restrict attention to households engaging on employer-employee working relationships. Because self-employment represents one extra margin for agents to avoid labor legislation, the results in this paper can be interpreted as a lower bound on the effects of the minimum wage on compositional changes between formal and informal sectors.

Minimum wage in Brazil. The minimum wage in Brazil is set by the federal government and imposes a nation-wide floor on monthly nominal earnings of formal workers in all sectors and occupations (Lemos, 2004). This wage is reviewed and adjusted annually, taking into account factors such as inflation and economic growth. Since the passage of the Lei Complementar No. 103 on July 14, 2000, five out of the 27 states have instituted state-specific wage floors: Rio de Janeiro and Rio Grande do Sul since 2001, Paraná since 2006, São Paulo since 2007, and Santa Catarina since 2010. However, these state-specific wage floors are not considered in the analysis, as

[7] In Figures A.1 and A.2 in online Appendix A, I compare the percentiles of the formal and informal earnings distribution in PNAD with those at RAIS and ECINF data sets, respectively, and confirm that PNAD is indeed a unified data set that provides a realistic picture of the earnings distribution in both sectors.

[8] Figure A.3 in online Appendix A shows that throughout the sample period less than 5% of workers had more than one job. This pattern holds true for both formal and informal workers.

[9] Engbom and Moser (2021) show that contracted hours in the formal sector are substantially concentrated on 44 hours per week (Figure B.25) and do not respond to changes in the minimum wage (Table 3). Moreover, the PNAD survey does not have information on contracted hours, the margin over which the minimum wage actually applies to. Hence, I decide to abstract away from that margin of adjustment.

Table 1: Summary statistics by formality status

	1996		2012	
	Formal	Informal	Formal	Informal
Share	60.9	39.1	69.1	30.9
Mean earnings	1,387	673	1,388	840
Share with HS	31.5	14.6	61.2	38.4
Age	32.5	31.0	33.7	33.5
Male	63.8	55.2	58.6	50.0

Notes: The first row displays the share of formal workforce in 1996 and 2012. The last four rows calculate means of the variables in the first column across formal and informal workers in 1996 and 2012. Earnings are deflated by CPI and expressed in 2012 values. *Sources:* 1996/2012 PNAD.

the federal minimum wage remains the most important wage floor for most of the population.

1.2 Stylized facts on the informal sector, inequality, and the minimum wage

This section establishes stylized facts on the informal sector, earnings inequality, and the minimum wage. First, I show that informal workers constitute a substantial share of the labor force, that these workers tend to earn less than workers in the formal sector, and that they are substantially less educated. Second, I show that earnings inequality in the informal sector did not fall throughout the sample period, differently from formal sector and overall inequality. Third, I highlight the substantial increase in the restrictions imposed by the minimum wage on the formal sector over the 2000s.

Facts on the informal sector. Table 1 calculates summary statistics by formality status in 1996 and 2012. The first row shows that, on average, 35% of the labor force is hired informally. The second row shows that earnings in the informal sector are, on average, 47% lower than formal sector earnings. The third row highlights that formal workers are more educated: there is a 20 p.p. gap in the share of workers with at least high school education in the formal versus informal sectors. The last two rows show that, relative to the formal sector, the informal sector has workers that are younger and has a larger share of female workers.

Table 1 also highlights substantial movements in the informal share and educational composition over time. The first row shows that the informal share fell from 39.1% to 30.9% between 1996 and 2012. At the same time, the third row displays substantial improvements in the educational levels of the Brazilian labor force in both sectors. Between 1996 and 2012, the share of workers with at least a high school diploma went up from 31.5% to 61.2% in the formal sector and from 14.6% to 38.4% in the informal sector. Figure A.4 in the online Appendix performs a shift-share analysis on the informal share of labor across different educational groups. It finds that aggregate improvements in education are key in explaining the overall reduction in informality (Haanwinckel and Soares, Forthcoming). This finding highlights the importance of controlling for changes in skill composition when assessing the effects of the minimum wage on the overall economy.

I now provide empirical support for two assumptions made throughout this paper: I abstract away from industry heterogeneity and the unemployment margin of adjustment. Table A.1 in the online Appendix shows that informality is widespread across different industries in Brazil. The within industry share of informal workers ranges from 17% in Manufacturing to 70% in Domestic Services. Moreover, Figure A.5 in the online Appendix performs a shift-share analysis and shows that what drove the reduction in the aggregate share of the informal sector was the reduction in informality levels within industries, rather than movements in the industry composition of the labor force. Figure A.6 in the online Appendix shows that, relative to the informal share of labor, the unemployment rate was much lower (around 7.5%) and presented lower variation throughout the sample period (9% to 6%). These are a consequence of the selection of workers that are highly attached to the labor force, and it implies a lower importance of the unemployment margin when compared to the informal margin of adjustment, the focus of this paper.

Facts on inequality. Figure 1 plots the evolution of the variance of log earnings in the aggregate, and broken down by formality status. There was a strong and steady reduction in overall (2.3% per year) and formal (4.5% per year) inequality in log earnings. At the same time, throughout the sample period, the variance of log earnings in the informal sector did not change much, fluctuating around 0.65 log points. Figure 1 also shows that throughout the 2000s there was a consistent widening in the gap between formal and informal earnings inequality.

I now show that inequality of earnings within the formal and informal sectors

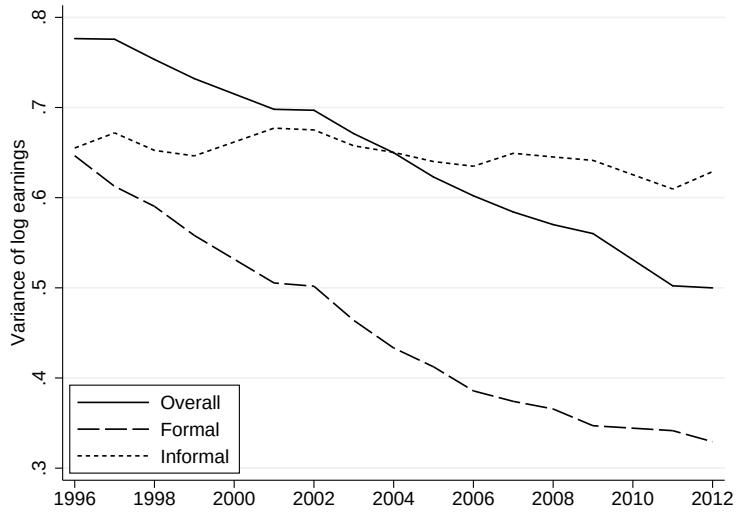


Figure 1: Variance of log earnings, 1996-2012

Notes: Variance of overall, formal, and informal log earnings between 1996 and 2012. *Sources:* 1996-2012 PNAD.

are the most important components of overall inequality. I decompose aggregate inequality into two parts: an employment-weighted average of inequality levels within the formal and informal sectors (the within component) and an employment-weighted sum of the squared distances between mean earnings in each sector and overall mean earnings (the between component):

$$V_t = \underbrace{\sum_{j \in \{form,inf\}} s_{jt} V_{jt}}_{\text{Within}} + \underbrace{\sum_{j \in \{form,inf\}} s_{jt}(E_{jt} - E_t)^2}_{\text{Between}} \tag{1}$$

Figure A.7 in the online Appendix plots the inequality decomposition over the sample period. The weighted sum of inequality within formal and informal sectors explains over 80% of the level of aggregate earnings inequality, and over 83% of the its reduction over the sample period.

Facts on the minimum wage. Figure 2 depicts two measures for the evolution of the minimum wage throughout the sample period. The left y-axis shows that the minimum wage as a fraction of median earnings increased from 45% in 1996 to 73% in 2012. At the same time, the share of minimum wage workers in the formal sector

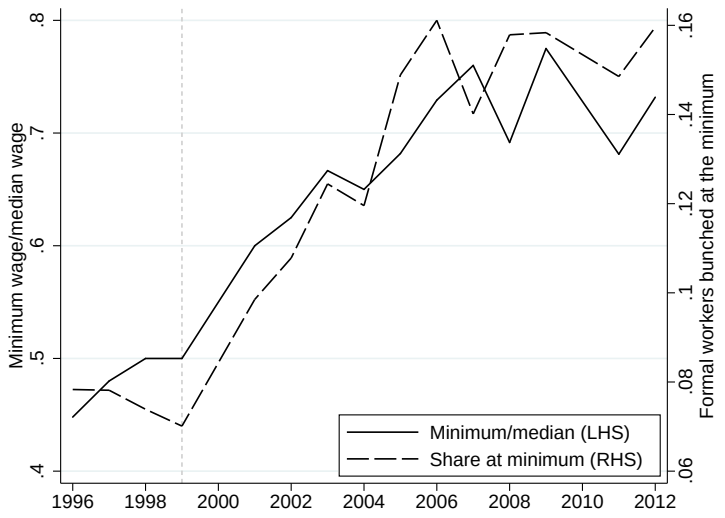


Figure 2: The evolution of the minimum wage, 1996-2012

Notes: Solid line shows the fraction of median earnings that the minimum wage represents (left axis). Long dashes denote the share of formal workers that receive exactly the minimum wage (right axis). *Sources:* 1996-2012 PNAD.

(right y-axis) increased substantially from 8% in 1996 to 16% in 2012. Importantly, the figure highlights that the bulk of the increase in the restrictions imposed by the minimum wage comes after 1999 - before that year there was a small decrease in the share of minimum wage workers in the formal sector.

1.3 Reduced-form evidence: minimum wage, inequality, and informality

In this section, I leverage state-level heterogeneity in exposure to the minimum wage to assess its effects on inequality and informality. I find that, relative to states least exposed to the minimum wage, states most exposed experienced decreases in formal inequality, increases in informal inequality, increases in the informal share of labor and, as a consequence, increases in overall inequality. These results suggest that the minimum wage can increase overall inequality despite decreasing inequality in the formal sector, due to its effects on the informal sector.

I start the analysis by ranking Brazil's 27 states according to their initial exposure to the minimum wage, measured by their 1999 share of formal workers binding at the national wage floor. I split them into 9 treatment groups, and compare the

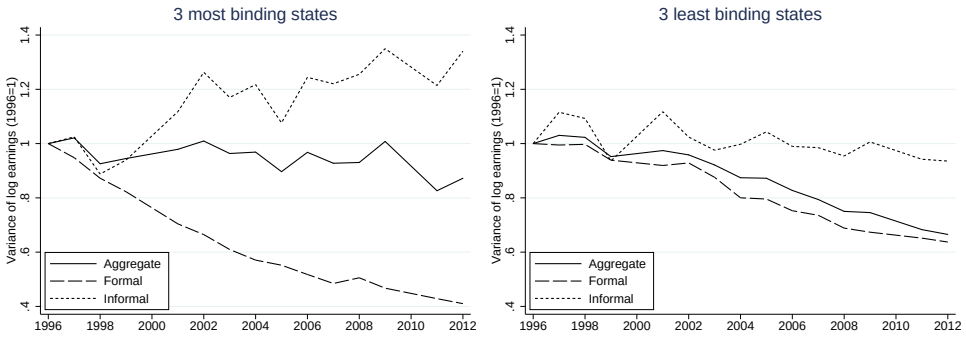


Figure 3: Earnings inequality in states most and least exposed to the minimum wage

Notes: Evolution of earnings inequality (1996 normalized to 1) in the formal and informal sectors, and in the aggregate. The plot on the left displays employment-weighted averages across the 3 states most binding (Piauí, Sergipe, and Bahia). The plot on the right displays employment-weighted averages across the 3 states least binding (São Paulo, Santa Catarina, and Distrito Federal). Exposure to the minimum wage is measured by the share of formal minimum wage workers. *Sources:* 1996-2012 PNAD.

evolution of formal, informal, and overall inequality, as well as the informal share of labor between the 3 most (Piauí, Sergipe, and Bahia) and 3 least (São Paulo, Santa Catarina, and Distrito Federal) exposed states.[10] The results of this exercise are displayed in Figures 3 and 4.

Figure 3 compares the evolution of aggregate (solid line), formal (long dashes), and informal (short dashes) earnings inequality in states most (left panel) and least (right panel) restricted by the minimum wage in 1999. Relative to least exposed states, the states that were most exposed to the minimum wage experienced stronger declines in formal inequality (60% versus 40%), a 40% increase in informal inequality, and milder declines in overall inequality (10% versus 40%). At the same time, Figure 4 compares the evolution of the informal share of labor in most (solid line) and least (long dashes) exposed states, and finds that states with the largest share of formal minimum wage workers experienced milder decreases in informality (13.6% versus 33.6%).

In what follows, I show that the patterns observed in Figures 3 and 4 are robust to controlling for differences across states, as well as other time-varying forces that are not directly related to the restrictions imposed by the minimum wage. I do so

[10] Table A.2 displays the full lists of states and groups.

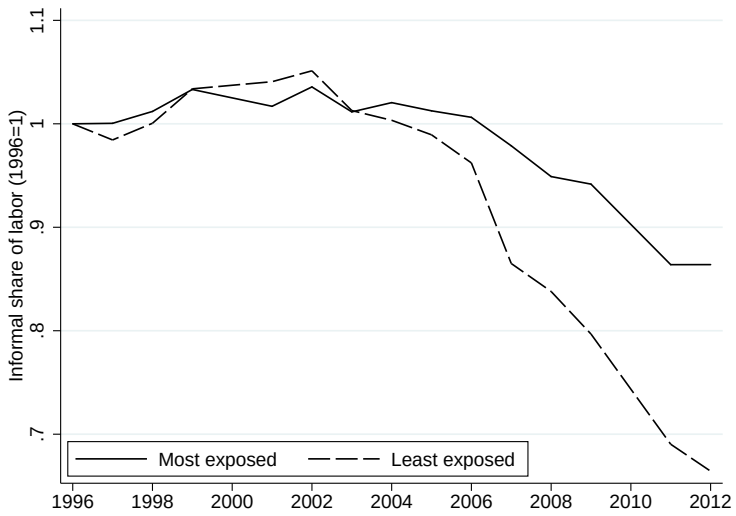


Figure 4: Informal share in states most and least exposed to the minimum wage

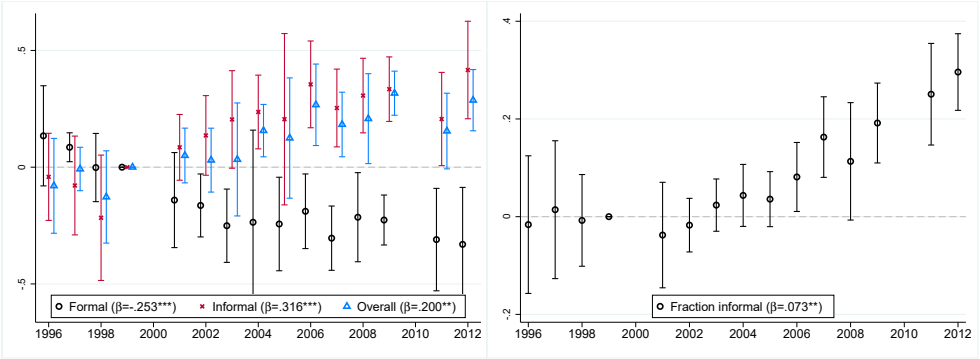
Notes: Evolution of the informal share of labor (1996 normalized to 1) in the most restricted states (solid line) versus least restricted states (long dashes). Exposure to the minimum wage is measured by the share of formal minimum wage workers. *Sources:* 1996-2012 PNAD.

via an event-study estimation strategy, where I interact a state's initial exposure to the minimum wage (the 9 groups constructed above) with year fixed effects:

$$y_{sgt} = \alpha + \sum_{h \neq 1} \sum_{k \neq 1999} \beta_{kh} \cdot \mathcal{I}_{g=h} \cdot \mathcal{I}_{t=k} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}, \tag{2}$$

where y_{sgt} denotes the outcome of interest in state $s = 1, ..., 27$, treatment group $g = 1, ..., 9$, and year $t = 1996, ..., 2012$, α is a constant, $\mathcal{I}$ are indicator functions, δ_s are state dummies, δ_t are year dummies, and X_{st} controls for the age, gender, race, and education compositions of the labor force, which might affect inequality and informality without necessarily being related to restrictions imposed by the minimum wage. I leave out the year of 1999, where minimum wage restrictions started to increase, as well as group 1, so that the estimates β_{kh} compare the evolution of outcomes in group h relative to the three states with the lowest share of formal minimum wage workers in 1999. The identification assumption is that of parallel trends: absent the sharp increase in the minimum wage after 1999, the relative evolution of outcomes in states belonging to different treatment groups would not change.

I first discuss the results for the most treated states (β_{k9}), staying as close as



(a) log(Variance) (b) log(Informal share)

Figure 5: Minimum wage, inequality, and the informal sector (most vs. least binding states)

Notes: This figure plots the ordinary least squares coefficients of Equation (2) for the states in the most treated group, β_{k9} for $k \neq 1999$. Panel (a) displays the results for outcomes related to earnings inequality. Panel (b) displays the results for the log of the informal share. The coefficients displayed in the legends correspond to the mean effect in the post-1999 period, and are displayed in Table 2. Standard errors are clustered at the state level. ***p<1%, **p<5%, *p<10%. *Sources:* 1996-2012 PNAD.

possible to the analysis in Figures 3 and 4. Figure 5 displays the annual coefficients for different outcomes. Panel (a) shows the results for the log of variance in the formal sector (black circles), informal sector (red crosses), and in the aggregate (blue triangles). Panel (b) displays the result for the log of the informal share. The corresponding average effects in the post-1999 period are displayed in Table 2. The results show that states most exposed to the minimum wage experienced a 25.3 p.p. stronger reduction in formal inequality, a 31.6 p.p. larger increase in informal inequality and a 7.3 p.p. stronger increase in the informal share of labor, which jointly led to a 20 p.p. stronger increase in overall inequality compared with least exposed states. Moreover, the coefficients associated with the years before 1999 provide supporting evidence for the identification assumption of parallel trends.

What about the other treatment groups? Figure 6 displays the mean effect of the minimum wage in the post-1999 period (y-axis) for each treatment group (x-axis). Panel (a) shows the results for overall (black circles), formal (red crosses), and informal (blue triangles) inequality, and Panel (b) plots the coefficients for the informal share. Three patterns stand out. First, minimum wages are effective in reducing inequality in the formal sector, with its effects becoming stronger in states that are more exposed. Second, minimum wages are associated with significant

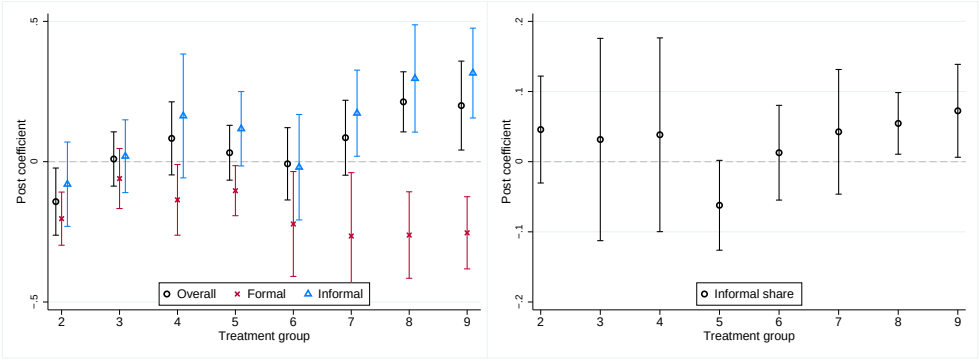
Table 2: Minimum wage, inequality, and the informal sector (mean effects post-1999)

	$\log(V^{\text{All}})$	$\log(V^F)$	$\log(V^I)$	log(Inf Share)
β_9	0.200	-0.253	0.316	0.073
	(0.077)**	(0.063)***	(0.078)***	(0.032)**
Fraction high skill	0.485	0.447	0.582	-0.466
	(0.177)**	(0.373)	(0.254)**	(0.106)***
Fraction under 30	-0.561	-0.511	-0.742	0.219
	(0.165)***	(0.320)	(0.198)***	(0.143)
Fraction white	-0.083	-0.233	-0.200	0.052
	(0.165)	(0.204)	(0.172)	(0.064)
Fraction female	0.218	0.446	0.558	-0.059
	(0.147)	(0.269)	(0.220)**	(0.226)
Unemployment rate	-0.343	-1.689	-0.042	0.253
	(0.521)	(0.723)**	(0.668)	(0.336)
State FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Observations	405	405	405	405
R^2	0.854	0.891	0.642	0.966

Notes: This table displays the coefficients of the OLS regression: $y_{sgt} = \alpha + \sum_{h \neq 1} \beta_h \cdot \mathcal{I}_{g=h} \cdot \mathcal{I}_{t>1999} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}$. The coefficients β_2-β_8 can be found in Figure 6 or in Table A.3 in the online Appendix. Standard errors in parentheses are clustered at the state level. ***p<1%, **p<5%, *p<10%. *Sources:* 1996/2012 PNAD.

increases in informal inequality and the evolution of informal share only in the most restricted states (groups 8 and 9). Third, and as a consequence, the effects of the minimum wage on overall inequality changes sign as you analyze states that are more or less treated. For example, relative to group 1, states in group 2 experienced a 14.2 p.p. stronger *decrease in overall inequality*, whereas states in groups 8 and 9 experienced a 20 p.p. stronger *increase in overall inequality.*

All in all, these results suggest that increasing the minimum wage increases overall earnings inequality whenever there is a strong, inequality-increasing effect on the informal sector. It is important to point out that the analysis in this section compares the evolution of outcomes in states more and less exposed to the minimum wage. Hence, it abstracts away from potential general equilibrium effects of the minimum wage, as these are captured by the control variables. These general



(a) log(Variance) (b) log(Informal share)

Figure 6: Diff-in-diff analysis (post-1999 effects across treatment groups)

Notes: This figure reports, for different treatment groups (x-axis), the coefficients of the OLS regression (y-axis): $y_{sgt} = \alpha + \sum_{h \neq 1} \beta_h \cdot \mathcal{I}_{g=h} \cdot \mathcal{I}_{t>1999} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}$. Panel (a) displays the results for outcomes related to earnings inequality. Panel (b) displays the results for the log of the informal share. The values for the coefficients can be found in Table A.3 in the online Appendix. Standard errors in parentheses are clustered at the state level. *Sources:* 1996-2012 PNAD.

equilibrium effects are accounted for by the quantitative model and counterfactual exercises studied in the next sections.

Robustness. I now discuss the robustness of the empirical findings to different specifications of informality and earnings. A first concern is that, by not including self-employed workers, my definition of informality does not capture all the margins of adjustment workers have away from the formal sector. To address this concern, I show that including self-employed individuals in the definition of informality does not alter the findings. Figures A.8 and A.9 highlight that the states most exposed to the minimum wage experienced increases in both informal and overall inequality, the opposite of what was experienced by states least exposed. Moreover, Figure A.10 replicates the event-study analysis including self-employed individuals and finds similar results to those in Figure 5. A second concern is the extent to which adjustment in hours worked could be influencing the results. I address this concern by considering hourly earnings as the measure for earnings. In this case minimum wage values are adjusted to a full-time, 44 hours per week working routine. Figures A.11-A.12 replicate the analysis in this case, and find similar results to that in the main specification, suggesting that adjustment in hours are not consequential.

I assess the robustness of the results to different regression specifications. First,

Figure A.13 shows that the overall patterns of the event-study analysis are the same if one splits states into two groups, according to whether their 1999 share of formal minimum wage workers are above or below the median. The fact that the estimates become noisier confirms our findings in Figure 6: the effects of the minimum wage on the informal sector are stronger in more exposed states. Second, Figure A.14 replicates the analysis in levels, and finds a positive and significant relationship between minimum wages and overall and informal earnings inequality and a positive but insignificant relationship between minimum wages and informal share. Third, there is a recent literature on two-way fixed effects models highlighting problems with ordinary least squares (OLS) estimators. Although the main concern is when there is staggered treatment dates, which is not the case in my setting (all states are subject to the post-1999 minimum wage increase), Figures A.15 and A.16 implement the estimators proposed in Borusyak et al. (2021), de Chaisemartin and D'Haultfœuille (2020), and Callaway and Sant'Anna (2020) and verify that the event-study coefficients are qualitatively and quantitatively similar to those in the main specification.

This section employs a differences-in-differences estimation strategy to correlate the federal minimum wage with inequality and informality. However, there is a large literature that correlates the minimum wage with earnings inequality via Kaitz regressions.[11] This literature uses the log-distance between the minimum wage and the median earnings in the formal sector (i.e., the Kaitz index) as a proxy for how stringent the minimum wage is in a state. Then, they correlate outcomes with the minimum wage by regressing the former on a quadratic specification of the latter, controlling for state and year effects. In online Appendix H, I closely follow this literature, detailing the regression specifications and identification assumptions, and show that my findings persist: minimum wages correlate negatively with formal inequality, positively with informal inequality and the informal share of labor, and these act as counteracting forces in the determination of the correlation between the minimum wage and overall inequality.

In a recent paper, Derenoncourt et al. (2021) provides evidence that the minimum wage in Brazil did not displace workers from the formal to the informal sector.[12] Although seemingly contradictory, the findings in this paper highlight the

[11] See Lee (1999), Autor et al. (2016), Engbom and Moser (2021), Haanwinckel (2020), and Urzua and Saltiel (Forthcoming).

[12] These findings are replicated in online Appendix J, where I divide the sample in two groups of states above and below median treatment.

importance of *treatment intensity* in evaluating the impact of the minimum wage on formal and informal employment. Figure 6 and the discussion in online Appendix J suggest that less-treated states did not experience a relative increase in the informal sector, and that the minimum wage displaces workers to the informal sector only in the poorest, most binding states in Brazil, like Piauí, Sergipe and Bahia.

2 Informality and the effects of the minimum wage

This section develops a stylized model to understand the effects of the minimum wage when the informal margin of adjustment is considered, before extending it in a number of directions in the quantitative analysis.[13] The model consists of ex-ante homogeneous workers and heterogeneous monopsonists that decide to operate formally or informally. Firms operating in the formal sector are subject to the minimum wage. Informal firms are not subject to the minimum wage, but are subject to government detection costs.

The model rationalizes the empirical findings from Section 1 on minimum wages, inequality and informality. For example, an increase in the minimum wage increases the size of the informal workforce, as well as earnings disparity between informal workers. Moreover, the effect of the minimum wage on overall inequality depends upon the strength of the informal margin of adjustment. When firm productivity is Pareto-distributed and there is no informal sector, increasing the minimum wage reduces overall inequality. However, when firms can avoid the minimum wage by operating informally, increasing the minimum wage *increases overall inequality*. Hence, the presence of an informal sector implies that there can be *unintended consequences of the minimum wage*: policies aimed to reduce inequality might end up increasing it because of the informal margin of adjustment.

2.1 Labor supply

There exists a unit measure of ex-ante homogeneous households. Each agent is endowed with one unit of time, supplied inelastically as labor. Households receive wage offers and must choose, after the realization of firm-specific amenity shocks, which firm to work for (Card et al., 2018). I assume that firm profits and government revenues are owned by households that consume the final good and do not participate

[13] All the derivations in the next two sections are detailed in online Appendix E.

in production.

The utility of an individual depends on their wages and the firm at which they work:

$$V_i(j) = A_i(j)w(j), \tag{3}$$

where $A_i(j)$ is an amenity shock household i gets for working in firm j, and $w(j)$ is their wage. I assume that $A_i(j)$ is independently distributed across households and firms, and drawn from a Fréchet distribution with shape parameter η.

The structure of the amenity shocks generates an upward-sloping labor supply curve at the firm level. Moreover, the law of large numbers implies that firm j's labor supply curve equals to the probability that household i optimally chooses to work for that firm:

$$l(j) = Prob_i(j) = \left[\frac{w(j)}{W}\right]^{\eta}, \tag{4}$$

with $W \equiv \left[\int_{j' \in \Omega} w(j')^{\eta} dj'\right]^{1/\eta}$ denoting the aggregate wage index and Ω denoting the exogenous set of operating firms.

2.2 Labor demand

There is an exogenous mass of firms that are ex-ante heterogeneous with respect to labor productivity. The productivity distribution follows $z \sim F$ over $[z_0 > 0, \infty)$, with $f(z) > 0$ for all z, and $lim_{b \to \infty} \int_{z_0}^{b} z^{\eta} dF(z) < \infty$. In the goods market, firms are perfectly competitive,[14] producing homogeneous goods that are perfect substitutes, which price is normalized to one. In the labor market, firms compete monopsonistically.

The timing of the problem of the firm is as follows. Conditional on productivity, firms decide on their formality status. In doing so, firms trade off minimum wages in the formal sector versus government detection costs in the informal sector. After the formality status is decided, firms maximize profits subject to the labor supply curve (4) and sector-specific constraints. At this stage, the monopsonistic competition assumption implies that larger firms must pay higher wages.

I start by calculating profits, employment, and wages conditional on the formality status. A firm with productivity z operating formally maximizes revenues net of

[14] Online Appendix D generalizes the results in this section for an environment of monopolistic competition and love for varieties.

labor costs, subject to the labor supply curve and the minimum wage ($\underline{w}$):

$$\pi^{form}(z) = \max_{\{l,w\}} \left\{ zl - wl \quad | \quad l = \left(\frac{w}{W}\right)^{\eta}, \quad w \geq \underline{w} \right\}. \tag{5}$$

Optimal wages, labor, and profits of the formal firm are:

$$w^{form}(z) = \max\left\{\frac{\eta}{\eta+1}z, \underline{w}\right\}, \quad l^{form}(z) = W^{-\eta}\max\left\{\frac{\eta}{\eta+1}z, \underline{w}\right\}^{\eta}, \tag{6}$$

$$\pi^{form}(z) = W^{-\eta}\max\left\{\frac{\eta}{\eta+1}z, \underline{w}\right\}^{\eta}\left[z - \max\left\{\frac{\eta}{\eta+1}z, \underline{w}\right\}\right]. \tag{7}$$

When unrestricted by the minimum wage, formal firms set wages as a markdown over the marginal product of labor. However, when the productivity of the firm is sufficiently small, the minimum wage becomes binding, and wages and labor no longer vary with firm productivity. Hence, the minimum wage operates as a fixed production cost for low productivity firms.

By operating informally, a firm is not subject to the minimum wage, but there is a penalty if it is detected by the government. Detection occurs with probability ρ, and the penalty is assumed to be a loss of all revenues.[15] I refer to ρ as the cost of informality. The problem of an informal firm with productivity z is:

$$\pi^{inf}(z) = \max_{\{l,w\}} \left\{ (1-\rho)zl - wl \quad | \quad l = \left(\frac{w}{W}\right)^{\eta} \right\}, \tag{8}$$

Optimal wages, labor, and profits of the informal firm are:

$$w^{inf}(z) = \frac{\eta}{\eta+1}(1-\rho)z, \quad l^{inf}(z) = W^{-\eta}\frac{\eta^{\eta}}{(\eta+1)^{\eta}}(1-\rho)^{\eta}z^{\eta}, \quad \pi^{inf}(z) = W^{-\eta}\frac{\eta^{\eta}}{(\eta+1)^{\eta+1}}(1-\rho)^{\eta+1}z^{\eta+1}. \tag{9}$$

Informal firms set wages as a markdown over the marginal product of labor. In this case, however, the marginal product of labor is affected by the cost of informality, as it scales down labor productivity. The absence of fixed costs implies positive profits for all firms in the informal sector. That is not the case in the formal sector: firms with productivity below the minimum wage have negative profits. Hence, informality acts as a profitable outside option for all firms in the economy.

[15] An alternative specification is that firms are detected with probability $\tilde{\rho} \in [0,1]$, in which case they lose a fraction $\gamma \leq 1$ of revenues. In this setting, expected revenues are $(1-\tilde{\rho})zl + \tilde{\rho}\gamma zl = [1-\tilde{\rho}(1-\gamma)]zl$. When $\rho = \tilde{\rho}(1-\gamma)$, revenues in this specification are the same as those in the main specification. Hence, changes in ρ reflect both changes in the probability of detection and changes in the share of revenue captured by the government upon inspection.

Conditional on productivity, firms choose the formality status that maximizes profits: $\pi(z) = \max\{\pi^{form}(z), \pi^{inf}(z)\}$. The corresponding labor demand and wages depend upon the formality decision, and are derived in Equations (6) and (9).

2.3 Equilibrium

This section defines and characterizes the equilibrium. An equilibrium is an aggregate wage index W such that aggregate labor supply equals aggregate labor demand:

$$L^D(W) \equiv \int_{z_0}^{\infty} l(z)f(z)dz = 1 = L^S. \tag{10}$$

The integral aggregates labor demand, $l(z)$, over all firms in the economy, weighted by their respective densities, $f(z)$. The last equality arises because aggregate labor supply is inelastic.

I now discuss firm selection into the informal sector conditional on the wage index (i.e., in partial equilibrium). Proposition 1 shows that the solution for the problem of the firm consists of two thresholds, $\underline{z} < \bar{z}$, in which firms with productivity below $\underline{z}$ operate informally, firms with productivity in $[\underline{z}, \bar{z}]$ are formal and restricted by the minimum wage, and firms above $\bar{z}$ operate formally and unrestricted by the minimum wage. Importantly, and in line with Machin et al. (2003), this stylized model environment is able to generate bunching of workers at the minimum wage, a realistic feature of the data.

Proposition 1. *There exists two thresholds:*

$$\underline{z}: \quad \frac{\eta^\eta}{(\eta+1)^{\eta+1}}(1-\rho)^{\eta+1}\underline{z}^{\eta+1} - \underline{w}^\eta \underline{z} + \underline{w}^{\eta+1} = 0 \quad \textit{and} \quad \bar{z} = \frac{\eta+1}{\eta}\underline{w} \tag{11}$$

such that:

1. $\underline{w} \leq \underline{z} < \bar{z}$;
2. *Firms with* $z < \underline{z}$ *operate informally, firms with* $z \in [\underline{z}, \bar{z}]$ *are formal but restricted by* $\underline{w}$*, and firms with* $z > \bar{z}$ *are formal and unrestricted by the minimum;*
3. $\frac{\partial \underline{z}}{\partial \rho} < 0, \quad \frac{\partial \underline{z}}{\partial \underline{w}} > 0, \quad$ *and* $\quad \frac{\partial^2 \underline{z}}{\partial \rho \partial \underline{w}} < 0$; *and*
4. $\frac{\partial(\underline{z}/\underline{w})}{\partial \underline{w}} = 0$

Proof. See online Appendix E for details. □

Why do unproductive firms select into the informal sector? When firms operate informally, they give up some productivity at the benefit of lowering labor costs. When productivity is low, the reduction in labor costs on minimum wage workers more than compensates for the losses in productivity. On the other hand, when firms are very productive, the productivity losses are too costly, so firms decide to comply with the minimum wage. The proposition also shows that larger minimum wages imply larger costs to operate formally, so a smaller share of firms will be productive enough to be formal. At the same time, smaller costs of informality compensate firms for being informal, increasing the share of firms that will optimally do so.

Proposition 1 also shows that the productivity cutoffs are independent of the aggregate wage index. This is because changes in the aggregate wage index do not alter relative profits across sectors. As a consequence, the equilibrium wage index is a markdown over the average productivity among all firms in the economy:

$$W = \frac{\eta}{\eta+1}\left[\int_{z_0}^{\underline{z}}[(1-\rho)z]^{\eta}f(z)dz + [F(\bar{z}) - F(\underline{z})]\,\bar{z}^{\eta} + \int_{\bar{z}}^{\infty} z^{\eta}f(z)dz\right]^{\frac{1}{\eta}}, \quad (12)$$

where the first term inside brackets represents the average productivity of informal firms, the second term represents the minimum wage constraints imposed to unproductive formal firms, and the last term represents the average productivity of the unconstrained, formal firms. The existence of the unique equilibrium is guaranteed by the assumption that $lim_{b\to\infty}\int_{z_0}^{b} z^{\eta}dF(z) < \infty$, so that the wage index is finite.

2.4 Inequality, minimum wage, and the informal sector

This section studies the effects of the minimum wage on earnings inequality, and how they change when the informal margin of adjustment is considered. To do so, I compare the effects of the minimum wage in an economy without informality ($\rho = 1$), versus an economy where firms can be informal ($\rho < 1$). Moreover, I assume that minimum wages are small: a marginal increase in the minimum wage generates the first units of informal labor in the economy with $\rho < 1$. Under these assumptions, Proposition 2 shows that under the presence of the informal sector, a minimum wage policy may have *unintended consequences*: an increase in the minimum wage can increase overall inequality.

Proposition 2. *Assume that the minimum wage ($\underline{w}$) is such that $\underline{w} \in (\underline{w}_0, \underline{w}_0 + \varepsilon)$, where $\underline{w}_0 : \underline{z}(\underline{w}_0) = z_0$ and small ε. Then, the marginal effect of the minimum wage on the variance of log earnings (V) is:*

$$\frac{\partial V}{\partial \underline{w}} = \underbrace{\frac{\partial V^{form}}{\partial \underline{w}}}_{\text{formal sector response (FR)}} + \underbrace{\underbrace{\frac{\partial L^{inf}}{\partial \underline{w}}}_{\text{workers become informal}} \left[\underbrace{\left(E^{inf} - E^{form}\right)^2}_{\text{wage differential}} + \underbrace{V^{inf} - V^{form}}_{\text{pre} \uparrow \underline{w} \text{ variances}} \right]}_{\text{informal sector response (IR)}} \tag{13}$$

If $z \sim Pareto(\nu > \eta)$,

1. *Without informality, increasing the minimum wage reduces inequality: $\frac{\partial V}{\partial \underline{w}} = \frac{\partial V^{form}}{\partial \underline{w}} < 0$.*
2. *With informality, increasing the minimum wage increases inequality: $\frac{\partial V}{\partial \underline{w}} > 0$.*

Proof. See online Appendix E for details. □

I first discuss the results in Proposition 2 for a general productivity distribution. When informality levels are low, the marginal effect of the minimum wage on the variance of log earnings can be decomposed into two parts. The first component represents the effects of minimum wages on the formal sector earnings inequality. This term is typically negative, and is the object of study of the aforementioned minimum wage literature. The second component corresponds to the informal margin of adjustment. It depends on the responsiveness of the informal sector to the increase in the minimum wage, as well as how "spread apart" the earnings distribution in both sectors are. Hence, the net effect of the minimum wage on aggregate inequality is ambiguous, depending on which component dominates.

Importantly, when firm productivity is Pareto-distributed, the net effect of a marginal increase in the minimum wage on inequality is positive. In a model without informality, inequality goes down as the minimum wage increases. However, this is only part of the story. Workers that become informal spread out the earnings distribution, raising inequality levels above and beyond the inequality-reducing effects the minimum wage has in the formal sector.

2.5 The effects of the minimum wage on worker welfare

In this section, I study how the minimum wage affects workers' welfare, and how that is influenced by the possibility that firms operate informally. I begin by calculating worker welfare. Recall that profits and government revenues are owned by households that consume the final good but do not participate in production. Hence, the expected utility of workers in this economy is proportional to the aggregate wage index:

$$\mathbb{E}[U] = \Gamma\left(\frac{\eta-1}{\eta}\right) W. \tag{14}$$

where $\Gamma(\cdot)$ denotes the gamma function. Because aggregate labor supply is inelastic, policies that increase aggregate labor demand increase the wage index, improving worker welfare. Hence, aggregate labor demand will be the object of study throughout this section.

I now analyze how the effects of the minimum wage on worker welfare are shaped by the presence of an informal sector. I operate under the same set of assumptions as in the last section. Proposition 3 decomposes the effects of the minimum wage on worker welfare into two parts. The first component is the formal sector response. This component is a direct consequence of the effect of the minimum wage in reducing the monopsony power of firms: instead of setting wage as a markdown over marginal product of labor, they are obliged to set wages at the minimum wage, increasing labor demand. The second component represents the informal margin of adjustment. An increase in the minimum wage increases the informality cutoff. Hence, firms at the cutoff now become informal, resetting wages as a markdown over discounted productivity, and adjusting their labor demand downward.

Proposition 3. *Let $l^{inf}(z)$ and $l^{\underline{w}}$ denote labor allocation at informal and minimum-wage firms. Assume that the minimum wage ($\underline{w}$) is such that $\underline{w} < z_0 < \underline{z} < \bar{z}$. Then, the marginal effect of the minimum wage on labor demand (L^D) is:*

$$\frac{\partial L^D}{\partial \underline{w}} = \underbrace{\underbrace{[F(\bar{z}) - F(\underline{z})]}_{\text{Firms at MW}} \underbrace{\frac{\partial l^{\underline{w}}}{\partial \underline{w}}}_{L^D \text{ increase}}}_{\text{formal sector response (FR>0)}} - \underbrace{\underbrace{\left[l^{\underline{w}} - l^{inf}(\underline{z})\right]}_{L^D \text{ drop (MW}\to\text{inf)}} \underbrace{f(\underline{z})}_{\text{firms at cutoff}} \underbrace{\frac{\partial \underline{z}}{\partial \underline{w}}}_{\text{cutoff response}}}_{\text{informal sector response (IR>0)}} \tag{15}$$

If $z \sim Pareto(\nu > \eta)$ then

1. *Without informality, increasing minimum wage increases worker welfare: $\frac{\partial L^D}{\partial \underline{w}} >$*

0.

2. *With informality, increasing the minimum wage reduces worker welfare:* $\frac{\partial L^D}{\partial \underline{w}} < 0$.

Proof. See online Appendix E for details. □

Importantly, the responses of the formal and informal sectors counteract each other, so the net effect of a marginal increase in the minimum wage depends upon which dominates. If F is Pareto,[16] the informal margin of adjustment is too strong, as there are a lot of firms concentrated around the informality cutoff, and increasing the minimum wage reduces workers' welfare. The proposition highlights that, under the above assumptions, increasing the minimum wage would be welfare-improving for workers in a model without informality, so the presence of the informal sector fundamentally alters the welfare consequences of the minimum wage.

3 Quantitative extension

This section describes the extended model, used to quantify the general equilibrium effects of the minimum wage. Consistent with the empirical evidence for Brazil, the additional features are important in shaping informality, inequality, and the way in which minimum wages influence the economy. On the household side, I assume that workers differ in their skill level, and that formal wages may worth more (or less) than informal wages due to the valuation of labor legislation. On the firm side, I introduce imperfect substitution between workers of different skills, payroll tax rates, and allow for the possibility that the productivity distributions in the formal and informal sectors overlap.

3.1 Labor supply

There is a unit measure of workers that are now assumed to differ in their skill level. In particular, there are H different skill levels, and let N_h denote the fraction of workers that are of skill level h. I maintain the assumption that profits and government revenues are owned by households that consume the final good but do not participate in production.

[16] The assumption of firm productivity being Pareto-distributed is common in the strand of literature on informality with heterogeneous firms, like Ulyssea (2018) and Haanwinckel (2020). This motivates why in propositions 3 and 2 I analyze this special case.

The utility of worker i, of skill h, working at firm j is:

$$V_{ih}(j) = A_i(j) \cdot (1 + \varsigma_h(j))w(j), \tag{16}$$

where $\varsigma_h(j) = 0$ if firm j is informal and $\varsigma_h(j) = \varsigma_h$ if formal. This formulation allows for a wedge between the nominal and the perceived value of a worker's wage, which may occur due to the value accrued to different labor legislation (e.g., vacation stipend, unemployment and retirement benefits, among others). In other words, the quantitative model allows for the possibility that one dollar of formal earnings may be worth more (or less) than one dollar of informal earnings.

The structure of amenity shocks is the same, so the labor supply curve firm j faces in the market for skill h is:

$$l_h(j) = N_h \left[\frac{(1 + \varsigma_h(j))w_h(j)}{W_h} \right]^{\eta}, \quad W_h = \left[\int_{j \in \Omega} [(1 + \varsigma_h(j))w_h(j)]^{\eta} \, dj \right]^{\frac{1}{\eta}} \tag{17}$$

with W_h being the wage index for skill h. Under these assumptions, the welfare of a worker with skill h is proportional their respective wage index, W_h.

3.2 Labor demand

There is an exogenous mass of firms with heterogeneous labor productivity. I assume that productivity has two components, $z = \nu\theta$, each drawn independently from their respective distributions F_ν and F_θ. Labor markets are segmented by skill, and firms compete monopsonistically in each of them. They aggregate labor from different skills in a CES fashion to produce a single, homogeneous good, sold under perfect competition.

The timing of problem of the firm is as follows. First, firms draw ν. Conditional on ν, firms decide whether to be formal or informal. Formal firms are subject to the minimum wage and payroll taxes, applied to all workers. Informal firms are subject to government detection costs. After they decide on the formality status, firms draw θ (hence, z is realized). Conditional on z, firms maximize profits subject to skill-specific labor supply curves (17) and sector-specific constraints.

As in the last section, I start by discussing the problem of the firm conditional on the formality status and labor productivity z. A firm in the formal sector has

profits:

$$\pi^{form}(z) = \max_{\{l_h(z), w_h(z)\}_h} \left\{ z\ell - (1+\tau) \sum_h w_h(z) l_h(z) \right\} \tag{18}$$

$$\text{s.t.} \quad \ell = \left[\sum_h \xi_h(z) l_h(z)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad l_h(z) = N_h \left[\frac{(1+\varsigma_h) w_h(z)}{W_h} \right]^{\eta}, \quad w_h(z) \geq \underline{w} \quad \forall h \tag{19}$$

where τ is the payroll tax rate, ε is the elasticity of substitution across skills, and $\xi_h(z)$ represent skill-specific demand shifters. The demand shifters capture the skill bias in the production function, and will be allowed to change over time to capture skill-biased technological change.

Proposition 4 shows that the above problem has a unique solution, in which wages are either the minimum wage or they reflect a markdown over the marginal product of labor. It also delivers an efficient algorithm to find the solution for the problem of the formal firm when constrained by the minimum wage.

Proposition 4. *Conditional on the productivity z, there exists a unique solution to the problem of the formal firm. In this solution, wages are either constrained at the minimum, or reflect a markdown over the marginal product of labor.*

Proof. See online Appendix E for details. □

Informal firms are not subject to minimum wages nor payroll taxes. However, due to government detection, they lose all revenue with probability ρ. An informal firm with productivity z has profits:

$$\pi^{inf}(z) = \max_{\{l_h(z), w_h(z)\}_h} \left\{ (1-\rho) z\ell(z) - \sum_h w_h(z) l_h(z) \right\} \tag{20}$$

$$\text{s.t.} \quad \ell(z) = \left[\sum_h \xi_h(z) l_h(z)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad l_h(z) = N_h \left[\frac{w_h(z)}{W_h} \right]^{\eta} \quad \forall h = 1, ..., H \tag{21}$$

Proposition 5 details the closed-form solution for the informal firm's problem. Firms set wages as an adjusted markdown over marginal productivity. The adjustment term takes into consideration the demand coefficient in the production function, $\xi_h(z)$, the supply of a given skill, N_h, and the relative cost of that skill to the firm, $W_h/\mathbb{W}(z)$.

Proposition 5. *Informal profits are:*

$$\pi^{inf}(z) = \mathbb{W}(z)^{-\eta} \frac{\eta^{\eta}}{(\eta+1)^{\eta+1}} (1-\rho)^{\eta+1} z^{\eta+1} \tag{22}$$

and labor and wages for each skill $h = 1, ..., H$:

$$w_h^{inf}(z) = [\xi_h(z)^{\varepsilon}/N_h]^{\frac{1}{\eta+\varepsilon}} [W_h/\mathbb{W}(z)]^{\frac{\eta}{\eta+\varepsilon}} \frac{\eta}{\eta+1}(1-\rho)z, \quad l_h^{inf}(z) = N_h \left[\frac{w_h^{inf}(z)}{W_h}\right]^{\eta} \tag{23}$$

where $\mathbb{W}(z)$ denotes the cost index a firm with productivity z faces:

$$\mathbb{W}(z) \equiv \left[\sum_h \xi_h(z)^{\frac{\varepsilon}{\eta+\varepsilon}(1+\eta)} \left(W_h/N_h^{1/\eta}\right)^{\frac{\eta}{\eta+\varepsilon}(1-\varepsilon)}\right]^{\frac{\eta+\varepsilon}{\eta}\frac{1}{1-\varepsilon}} \tag{24}$$

Proof. See online Appendix E for details. □

I now discuss the formality decision of firms. Firms will operate formally if and only if their expected formal profits are larger than their expected profits of being informal:

$$V^{form}(\nu) \equiv \int_{\theta\in\Theta} \pi^{form}(\nu\theta) dF_\theta(\theta) \geq \int_{\theta\in\Theta} \pi^{inf}(\nu\theta) dF_\theta(\theta) \equiv V^{inf}(\nu) \tag{25}$$

This two-stage process has two implications that are worth discussing. First, it generates an overlap in the productivity distribution of firms in the formal and informal sectors, leading to an overlap in the wage distributions in the two sectors, a predominant feature in the data. Second, there is exit of formal firms that are not productive enough (low draw of θ) to operate with positive profits.

3.3 Equilibrium

An equilibrium is a set of wage indices W_h, for all $h = 1, ..., H$, where aggregate labor demand equals aggregate labor supply:

$$\int_0^{\infty} l_h(z) dF(z) = N_h, \quad \forall h = 1, ..., H \tag{26}$$

In online Appendix F, I calculate the market clearing condition in the goods markets, and online Appendix G details the algorithm used to numerically solve for the

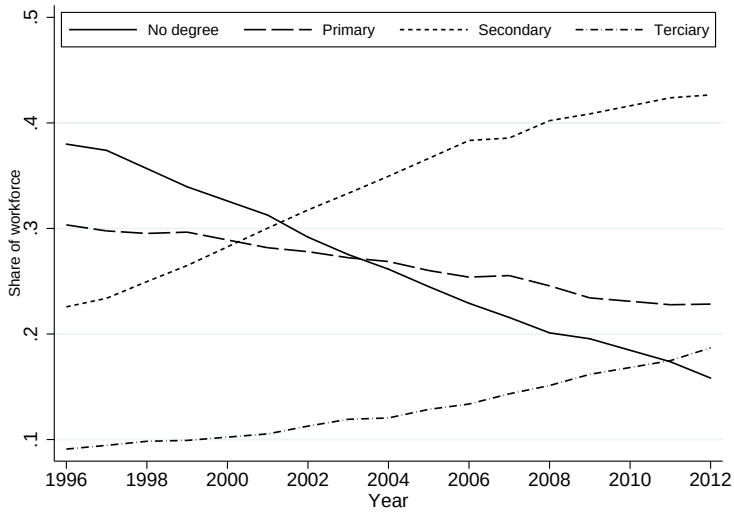


Figure 7: Skill composition of Brazilian labor force, 1996-2012

Notes: Share of labor force that belongs to each education group. *Sources:* 1996-2012 PNAD.

equilibrium.

4 Calibration and validation

This section calibrates the quantitative model to the Brazilian data. Unless noted, all parameters have 1996 and 2012 values. First, I discuss the calibration of the parameters associated with labor supply, labor demand, and government, respectively. I show that the values obtained for the parameters internally calibrated are in line with central values in the literature. Second, I perform external validation checks on the model's predictions by comparing moments not targeted in the data. In particular, I show that the model delivers realistic earnings distributions for the overall economy, within each sector, and within each skill group.

4.1 Labor supply

I map workers' skills to education in the data. I construct four relatively standard education groups ($H = 4$), depending on whether a worker does not have a degree (4 years of education or less), has a primary degree (5-8 years of education), has a secondary degree (9-11 years of education), or has a tertiary degree (over 12 years

of education). Figure 7 plots the share of workers of each skill between 1996-2012, which maps directly into N_h. There was substantial increase in levels of educational attainment in the Brazilian labor force, with the share of individuals with no degree (N_1) falling from 38% in 1996 to 16% in 2012.

The definition of skills above captures substantial heterogeneity in the earnings distribution. For example, Figure A.17 in the online Appendix plots the distribution of log earnings relative to the minimum wage for 1996 and 2012 across different skill groups. Even though there is substantial overlapping, workers with tertiary degrees earn on average 4 times more than non-degree workers. Importantly, Figure A.18 shows that the differences in mean earnings of higher educated workers relative to non-degree workers decreased substantially in the 2000s, a pattern that will be captured by the parametrization of the model.

I internally calibrate the elasticity of the firm-level labor supply curve to match the unconditional formal-informal mean wage gap. I obtain $\eta = 4.52, 4.22$ for 1996 and 2012, respectively. The identification of this parameter comes from the fact that η influences the slope of firm-level wages to firm-level productivity. When η is small, wages vary less with firm productivity, attenuating the impact of formal-informal firm selection on workers' equilibrium wages.[17] Importantly, the estimated values for η are relatively constant over time, and are in the same range of values estimated by the labor literature (Card et al., 2018; Lamadon et al., 2019).

4.2 Labor demand

I incorporate skill bias in the technology by assuming that more (less) productive firms are more intensive in skilled (unskilled) labor. In particular, I assume a structure for the demand shifters similar to that in Burstein and Vogel (2017):

$$\xi_h(z) = \frac{z^{\phi_h}}{\sum_j z^{\phi_j}}, \quad \sum_h \phi_h = 0. \tag{27}$$

When $\phi_h > 0$, more productive firms are more intensive in skill h. I internally calibrate these parameters, targetting the ratio of mean earnings across different skills relative to workers with no degree. Figure 8 displays the values obtained for 1996 and 2012. The fact that the demand coefficients for skill groups 3 and 4 increase

[17] For instance, in the stylized model wages were set as $w = \frac{\eta}{\eta+1} z$ so the slope of wages with respect to z depends directly on how large η is.

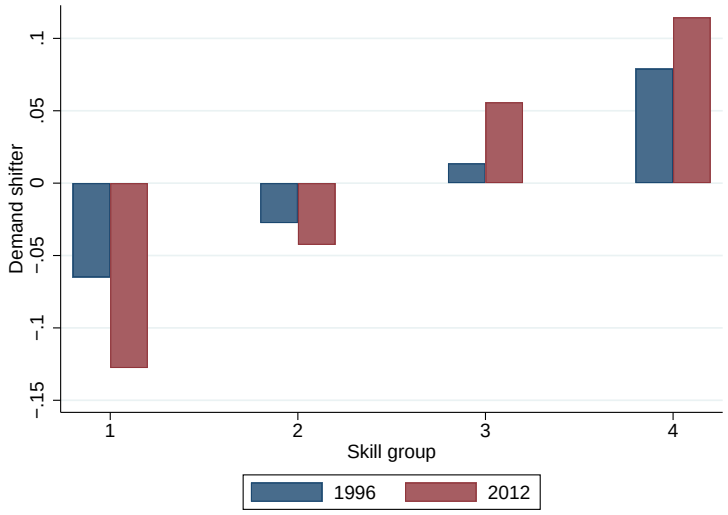


Figure 8: Calibrated demand shifter parameters (ϕ_h)

Sources: 1996-2012 PNAD and model simulations.

over time suggests that Brazil experienced skill-biased technical change throughout the 2000s, in line with Haanwinckel (2020) and others. In other words, the calibration results suggest that more (less) productive firms became more specialized in qualified (unqualified) labor over time.

I set a constant value of $\varepsilon = 1.875$ for the elasticity of substitution. This value lies in the range of elasticities of substitution between workers of different education levels Fernández and Messina (2018) estimates using data from Latin American countries (from 1.16 to 2.51).[18] Lastly, I assume that the first productivity component, ν, is drawn from a Log-Normal distribution where the underlying Normal has mean zero and standard deviation σ. The second component of productivity, θ, is drawn from a Pareto distribution with shape parameter κ. This delivers a Pareto-LogNormal distribution of firm productivity, first introduced in Colombi (1990) and used further in the literature (Rothschild and Scheuer, 2016; Ulyssea, 2018). I internally calibrate these parameters to match the variance of log earnings in the formal and informal sectors in each year, resulting in $\sigma = 1.01, 1.29$ and $\kappa = 6.02, 6.33$ for 1996 and 2012, respectively. Between 1996 and 2012, there was an increase in the dispersion of the base of the productivity distribution, as σ went up, and a decrease

[18] It is also in line with values found for the US literature (Katz and Murphy, 1992).

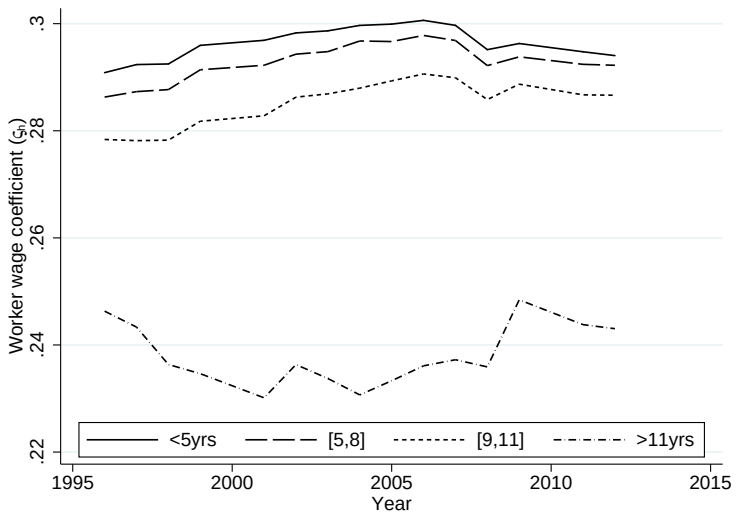


Figure 9: Workers' valuation of wages (ς_h), 1996-2012

Notes: Valuation of gross wages, estimated from labor legislation using the methodology in Souza et al. (2012). *Sources:* 1996-2012 PNAD and Brazilian labor legislation.

in the tail, as κ increased.

4.3 Government

The minimum wage and the informality cost are internally calibrated. I target the share of formal workers binding at the minimum wage and the overall size of the informal sector. The calibration process implies $\underline{w} = 4.04, 8.87$ and $\rho = 0.26, 0.32$ for 1996 and 2012, respectively. The model-implied 120% increase in the minimum wage is in line with the observed 106% increase in the real minimum wage. Moreover, the calibrated model delivers an increase of 23% in the informality cost. This is consistent with observed efforts by the Brazilian government to deter the growth in the informal sector and increase the enforcement of labor regulation (Corseuil et al., 2012).

The PNAD reports information on gross wages, and these are also the notion of wages in the model, over which the minimum wage operates. However, because of labor legislation, there is a disconnect between the perceived and the nominal value of formal wages, for both firms (τ) and workers (ς_h). I now discuss how I take this into account in the quantitative exercise.

Recall that I assume that the formal firm has a total labor cost of $1+\tau$ times its gross wage bill. This takes into consideration the fact that a firm must pay vacation stipends, social security contributions, severance payments, among other transfers, to its workers. In online Appendix B, I adapt the methodology from Souza et al. (2012) and estimate that, over the sample period of 1996-2012, $\tau = 71.4\%$. I assume this value to be constant over time once there was little changes in these transfers and contributions over the sample period.

Similarly, recall that each Brazilian Real a formal worker of skill h receives has a value of $1+\varsigma_h$. Online Appendix B details the labor legislation behind the estimation of ς_h, which also follows the methodology in Souza et al. (2012), and are displayed in Figure 9. The figure highlights two important findings. First, the fact that $\varsigma_h > 0$ for all h implies that the benefits accrued from vacation stipends, unemployment benefits, and others, more than compensates for the costs associated with income taxation and mandatory social security contributions. Second, the gap between the nominal and the real value of formal wages is 30% for no-degree workers and 24% for workers with tertiary education, this being a consequence of progressive taxation both in social security and income tax.

To summarize, Table 3 displays the parameters of the model, detailing the targets for those calibrated inside the model and sources used for those calibrated outside of the model. Figure A.20 in the online Appendix illustrates how changes in each parameter, around their calibrated values, affect the objective function. The results suggest that each parameter plays an important role in minimizing the distance between model and data moments.

4.4 Discussion and external validation

This section discusses how well the model fits the data along targeted and untargeted moments. Table 4 compares model and data moments on relative wages across skills/sectors, earnings inequality, minimum wage bindingness, and the informal sector. The calibrated model is able to replicate targeted moments well, with the exception of inequality in the informal sector. There is an inherent tension in the model between getting the correct inequality within sectors and the right size of the informal sector: in 1996, in order to generate similar levels of inequality in the formal and informal sector, one must have a relatively low share of informal labor, whereas in 2012 the opposite holds true - generating such discrepancy in inequality between sectors demands a larger share of informal labor.

Table 3: Parameters of the model

	Description	1996	2012	Target/Source
1. *Labor supply:*				
H	# of skills	4		Education groups
N_h	Skill supply	Figure 7		PNAD
η	Labor supply elast.	4.52	4.22	Formal wage premium
2. *Labor demand:*				
σ	z distribution	1.01	1.29	$V^{inf}(\log w)$
κ	z distribution	6.02	6.33	$V^{form}(\log w)$
ϕ_h	Demand shifters	Figure 8		Relative wages
ε	EoS across skills	1.875		Fernández and Messina (2018)
3. *Government*:				
$\underline{w}$	Min. wage	4.04	8.87	Share at min. wage
ρ	Inf. cost	0.26	0.32	Informal share
ς_h	Earnings tax	Figure 9		PNAD and legislation
τ	Payroll tax	71.4%		Legislation

Notes: The internal calibration procedure searches for $\Theta^* = \{\phi_1^*, \phi_2^*, \phi_3^*, \sigma^*, \kappa^*, \eta^*, \rho^*, \underline{w}^*\}$ that minimizes the mean absolute percentage distance between data and model moments: $\Theta^* = argmin_\Theta \sum_{i=1}^{8} |m_i(\Theta)/\hat{m}_i - 1|$, where $\hat{m}_i$ are the targeted moments and $m_i(\Theta)$ are their model counterparts. To find the global minimum, I first evaluate the objective function at different initial points, find the resulting local minima starting at each initial point, then compare the values across minima. I used the 1996 and 2012 values for N_h and ς_h, displayed as the initial and end points in Figures 7 and 9, respectively. *Sources:* 1996-2012 PNAD and model simulations.

The model does well in replicating other untargeted moments, such as inequality measures within different skill groups and the relative log distance between the minimum wage and mean earnings. On the other hand, there is less heterogeneity in the informal share of labor within each skill group in the model than in the data.

Lastly, I show that the calibrated model generates realistic earnings distributions. I do so by comparing data and model-generated histograms of log earnings relative to the minimum wage. Figure 10 displays histograms for the aggregate, formal, and informal distributions of earnings in 1996 (left column) and 2012 (right column). The top-most figures, which look at aggregate distributions, show that the model economy generates similar moments other than the mean and the variance. Moreover, the figure suggests that the Pareto Log-Normal assumption for the distribution of productivities, which ultimately shapes the wage distribution in the

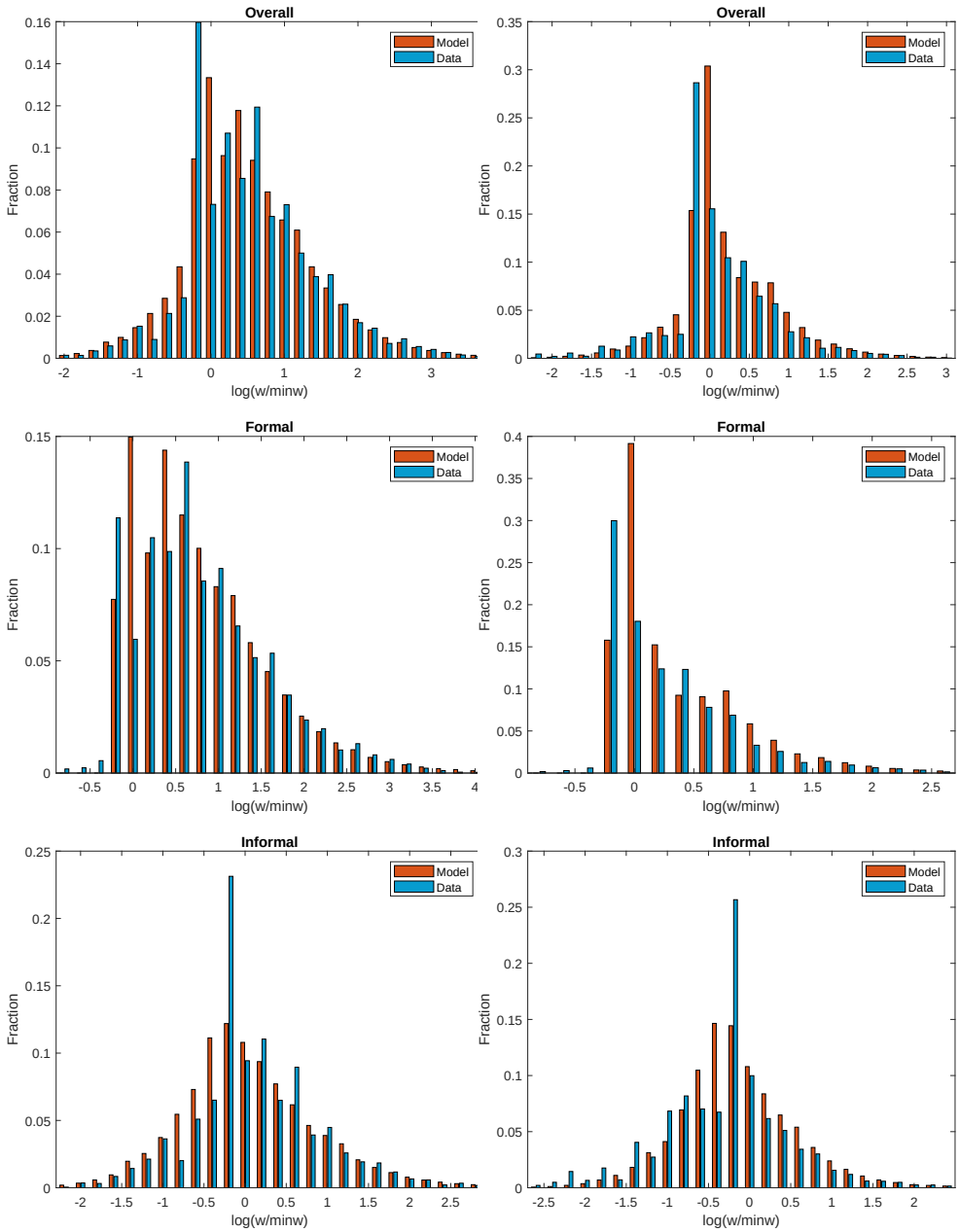
Table 4: Model moments

	1996		2012	
	Data	Model	Data	Model
1. *Mean earnings*:				
Formal/Informal	**2.06**	**2.11**	**1.65**	**1.67**
Primary/No degree	**1.39**	**1.39**	**1.19**	**1.19**
Secondary/Primary	**1.46**	**1.49**	**1.21**	**1.21**
Tertiary/Secondary	**2.49**	**2.41**	**2.15**	**2.15**
2. *Variance of log-earnings*:				
Overall	0.78	0.78	0.50	0.46
Formal	**0.65**	**0.58**	**0.33**	**0.33**
Informal	**0.66**	**0.73**	**0.62**	**0.51**
No degree	0.54	0.55	0.45	0.25
Primary	0.54	0.60	0.34	0.32
Secondary	0.64	0.63	0.32	0.37
Tertiary	0.91	0.69	0.64	0.40
3. *Minimum wage*:				
(Formal) Fraction at $\underline{w}$	**7.74**	**7.74**	**15.8**	**15.8**
(Formal) $\frac{\text{Min wage}}{\text{Mean wage}}$	0.22	0.26	0.45	0.47
4. *Informal share*:				
Overall	**0.39**	**0.39**	**0.31**	**0.31**
No degree	0.52	0.39	0.49	0.29
Primary	0.37	0.40	0.36	0.32
Secondary	0.26	0.39	0.23	0.31
Tertiary	0.22	0.37	0.22	0.31

Notes: Bold values are moments targeted in the calibration procedure. *Sources:* 1996-2012 PNAD and model simulations.

model, provides a good approximation for the lower and upper tails of earnings.

The bottom-most plots compare the earnings distribution within the formal and informal sectors in the model and data. The model is able to capture the correct bunching at the minimum wage in the formal sector, and it is again capable of generating realistic earnings distributions. The same is true for the informal sector, although there is less bunching at the minimum wage in the model than in the data,



(a) 1996 (b) 2012

Figure 10: Log earnings histogram, 1996-2012

Notes: Histograms of log earnings relative to the minimum wage. Widths are set to 0.2. Blue histograms are model, red are data. *Sources:* 1996-2012 PNAD and model simulations.

Table 5: Counterfactual exercise

		Counterfactuals				
	1996	$\underline{w}$ (min wage)	ρ (enforcement)	N_h (skill comp)	$\xi_h(z)$ (SBTC)	2012
Mean earnings						
prim/no deg	1.39	1.37	1.39	1.03	1.76	1.19
sec/prim	1.49	1.49	1.49	0.92	2.07	1.21
terc/sec	2.41	2.41	2.41	2.35	2.31	2.15
form/inf	2.11	1.70	2.37	2.52	1.90	1.67
V(log earnings)						
overall	0.78	0.83	0.78	0.79	0.98	0.46
formal	0.58	0.51	0.58	0.62	0.70	0.33
informal	0.73	0.83	0.72	0.67	0.94	0.51
Fraction at $\underline{w}$	7.74	15.2	8.33	3.66	14.0	15.8
Min/mean wage	0.26	0.33	0.26	0.21	0.24	0.47
Informal share	0.39	0.73	0.28	0.23	0.61	0.31

Notes: Each column under the Counterfactuals label changes one parameter from its 1996 calibrated level to its counterfactual levels. The 1996 and 2012 columns are replicated from Table 4. *Sources:* Model simulations.

a phenomenon highlighted in Derenoncourt et al. (2021). Lastly, Figure A.19 in the online Appendix analyzes the within-skill earnings distribution and shows that, yet again, the model economy reproduces realistic earnings distributions along this dimension.

5 Counterfactuals

This section evaluates the role of the minimum wage in shaping the aggregate distribution of earnings and the informal share of labor. The counterfactual minimum wage of $\underline{w} = 6.6$ is calculated by assuming that all of the increase in the bunching of formal workers between 1996 and 2012 — from 7.7% to 15.8% — was induced *solely* by the minimum wage increase. I also explore the extent to which formal enforcement, the skill composition and skill biased technical change affect inequality and informality, and might interact with the effects of the minimum wage. I do so

by changing each associated parameter, one at a time, to its estimated 2012 value, and solving for counterfactual aggregate outcomes. The main results are displayed in Table 5. The associated wage indices in 1996 and in each counterfactual scenario, which are proportionately related to worker welfare, are shown in Table 6.[19]

What are the effects of the increase in the minimum wage? The first three rows of the minimum wage column show that increasing $\underline{w}$ does not affect the skill earnings premia. On the other hand, when minimum wages go up, there is a substantial decrease in the formal wage premium, as it is shown in the fourth row. This is a consequence of productive formal firms becoming informal, coupled with a strong increase in the share of minimum wage workers. In line with the minimum wage literature, the increase in the minimum wage is responsible for 28% of the observed decrease in formal inequality (Engbom and Moser, 2021). Moreover, Figure 11 shows that the minimum wage increase has spillover effects up to the 75th percentile of the formal earnings distribution. All else equal, raising the minimum wage increases the p10p90 ratio by 21.8%, the p20p90 ratio by 12.4%, the p50p90 by 3.5% but has little effects beyond the median. These spillovers take place due to the imperfect substitution between workers of different skills, some of which are more exposed to the increase in the minimum wage.[20]

However, I find that the minimum wage increase in Brazil had the *unintended consequence* of increasing overall earnings inequality. That is, the increases in informality and informal inequality more than compensate for the inequality-reducing effect of the minimum wage on the formal sector, resulting in an increase in aggregate earnings inequality of 6.4%. Lastly, because of the strong informal response and the fact that informal wages are much smaller than their formal counterparts, increasing the minimum wage is welfare improving only for workers with tertiary education (0.7% increase in welfare).

I now analyze what fraction of the labor force works for firms that: (1) cannot cope with the 2012 minimum wage (type 1); or (2) are productive enough to be formal under the 2012 minimum wage, but optimally chose operate informally (type

[19]The finding that the minimum wage increase raised overall earnings inequality is robust to other counterfacutal experiments. For example, unintended consequences are also present in a counterfactual that increases the 1996 minimum wage until the share of formal minimum wage workers equals the one observed in 2012. In this experiment, the counterfactual minimum wage goes up by 41.6%, overall earnings inequality goes up by 5.1% and 69.1% of the labor force becomes informal. Further results for this alternative counterfactual are available upon request.

[20]This is in line with the findings in Engbom and Moser (2021), which show substantial spillover effects of the minimum wage along the formal wage distribution in Brazil.

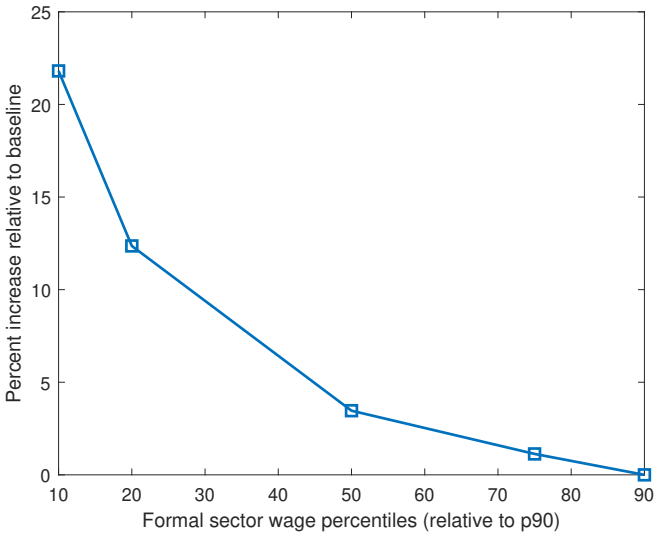


Figure 11: The effects of the minimum wage along the formal earnings distribution

Notes: Percent increase in different percentile ratios, relative to the 1996 baseline. All percentile ratios are relative to the 90th percentile. *Sources:* Model simulations.

Table 6: Welfare

Wage indices (W_h)	1996	$\underline{w}$	ρ	N_h	$\xi_h(z)$
No degree	0.967	0.958	0.942	1.618	0.690
Primary	1.240	1.235	1.200	1.564	1.066
Secondary	1.730	1.727	1.680	1.358	1.902
Tertiary	3.687	3.712	3.593	2.733	3.952

Notes: This table plots the equilibrium wage indices expressed in Equation 26 in the 1996 baseline economy and each counterfactual scenario. *Sources:* Model simulations.

2). I find that 25.8% of workers are employed in type-1 firms, and 9.9% of the labor force works in firms of type 2. Hence, 27.7% of the labor force that becomes informal in response to the minimum wage work in firms that could be formal if they were forced to do so. These results suggest that formalization policies, if implemented in companion with minimum wage adjustments, have the potential to deter a large share of the increase in informality without leading to firm exit.

What are the economic effects of the other mechanisms displayed in Table 5?

Table 7: Joint counterfactuals: minimum wages, formal enforcement and the skill composition

		Counterfactuals			
	1996	$\underline{w}$	$+\Delta\rho = 24\%$	$+\Delta\rho = 85\%$	$+\Delta N_h$
V(log earnings)					
overall	0.78	0.83	0.84	0.78	0.77
formal	0.58	0.51	0.50	0.49	0.54
informal	0.73	0.83	0.81	0.78	0.74
Fraction at $\underline{w}$	7.74	15.2	15.8	17.8	7.50
Informal share	39.1	72.8	57.6	23.6	47.4

Notes: This table shows the counterfactual effects of a joint change in the minimum wage and the informality cost, and the minimum wage with the skill composition. The 1996 and $\underline{w}$ columns replicate the results in Table 5. The $\Delta\rho = 24\%$ column evaluates the effects of changes in both the minimum wage and the estimated increase in the informality cost. The $\Delta\rho = 85\%$ column analyzes the joint effects of the minimum wage and a 85% increase in informality costs relative to its 1996 estimated value. The ΔN_h analyzes the joint effects of the minimum wage increase and the improvement in the skill composition. *Sources:* Model simulations.

The increase in informality costs are associated with a 28% decrease in the share of informal labor. On top of that, there is no change in aggregate earnings inequality. This result stems from two counteracting forces: on the one hand, there is a strong increase in inequality across sectors, captured by the 12.3% increase in formal earnings premium. On the other hand, there is a decrease in the informal inequality. These effects net out and there is little change in overall inequality.

Is the estimated increase in formal enforcement enough to compensate for the unintended consequences of the minimum wage? Table 7 displays the effects of the minimum wage when analyzed on its own, as well as in conjunction with changes in the informality cost ρ. The estimated increase of 24% in the informal costs does little in preventing the minimum wage from triggering a large share of informal workforce and increasing the overall earnings inequality. However, the second-to-last column suggests an increase of 85% in formal enforcement between 1996 and 2012 would have offset the effect of the minimum wage on aggregate inequality. These results highlight the importance of formalization policies to take place in conjunction with changes in the minimum wage to prevent them from having unintended consequences on aggregate inequality.

The improvement in the skill composition is an important driver of the reduction in the share of informal labor, reducing it by 30% (Haanwinckel and Soares, Forthcoming). Low productivity firms select themselves into the informal sector (Ulyssea, 2018). At the same time, these firms are more intensive in low-skilled workers. This factor of production, however, becomes more scarce, its wages go up, and operating informally becomes relatively more expensive. As more firms become formal, there is an increase of 6.4% in formal inequality, which is compensated by the 11.1% decrease in informal inequality, leaving aggregate inequality nearly unaffected. The welfare effects of the change in the skill composition are intuitive: factors that become relatively more scarce (no-degree and secondary education workers) see an increase in welfare vis-a-vis factors that become more abundant. Moreover, the last column in Table 7 shows that improvements in the skill composition can also complement minimum wage policies in reducing overall earnings inequality. By substantially reducing the size of the informal sector and shifting the formal labor force away from the binding minimum wage, the improvement in the quality of the labor force allows more workers to benefit from the minimum wage increase, alleviating the unintended consequences of this labor market policy.

Lastly, I analyze the effects of the skill-biased technical change. The shift in demand towards workers with secondary and tertiary education improves their welfare by 10% and 7.2%, respectively, at the expense of decreases of 29% and 14% in welfare for no degree and primary education workers (Table 6). The SBTC increases the informal share of labor from 39% to 61%. As low productivity firms become more intensive in the factors that are the cheapest, there is an increase in expected informal profits, inducing formal firms at the margin to switch formality status. The change in the demand coefficients increase aggregate earnings inequality by 26%. This is a result of increases in inequality within both sectors; informal sector inequality expands as a direct consequence of a wider range of firms becoming informal, whereas formal sector inequality expands as there is an increase in the distance of wages paid by unconstrained and minimum wage firms.

Unemployment. Online Appendix I adds the unemployment margin to the quantitative model, following Caliendo et al. (2019). I re-calibrate the model's parameters, including the one that governs unemployment benefits. After performing a similar set of counterfactuals, I find that the most important margin of adjustment in response to the minimum wage increase is between the formal and informal sec-

tors, not between employment and unemployment. This stems from the relative distance between the calibrated unemployment benefits and minimum wage, and how they place along the earnings distribution.

6 Conclusion

There is a long-standing literature suggesting that minimum wages are an important tool for reducing earnings disparity. In this paper, I examine how this effect is shaped by the presence of the informal sector. I find that, in the Brazilian context, the spike in the minimum wage over the 2000s actually increased overall inequality, highlighting the *unintended consequences* of the minimum wage. That is, policies that aim at reducing inequality might end up increasing it due to strong informal margins of adjustment.

I reach this conclusion in three steps. My empirical work provides reduced-form evidence that the minimum wage increases inequality in the informal sector, and that this offsets the inequality-reducing effects the minimum wage has in the formal sector. I then devise a theoretical model and derive sharp analytical results showing that there is scope for higher minimum wages to increase aggregate earnings inequality. In the last step, I build a quantitative framework to study the role of changes in the minimum wage, formal enforcement, skill composition, and skill-biased technical change on the Brazilian economy. I show that the minimum wage increase, albeit responsible for a strong reduction in formal sector inequality, is also responsible for an increase of 6.4% in aggregate inequality, due to a strong informal margin of adjustment.

This paper opens important avenues for further research. First, it provides a tool for addressing the discussion about federal-level minimum wages in countries where local labor markets differ substantially in informality levels. Second, as alternative work arrangements take place in developed countries (e.g., Uber/Lyft drivers and Grubhub/Doordash deliverers), the question of how to properly assess the effects of the minimum wage when agents can contract outside of labor legislation becomes of first order. All in all, my findings suggest that movements into and out of the informal sector modulate the effects of formal labor legislation in developing countries.

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Minimum Wages, Inequality, and the Informal Sector

Rafael Machado Parente

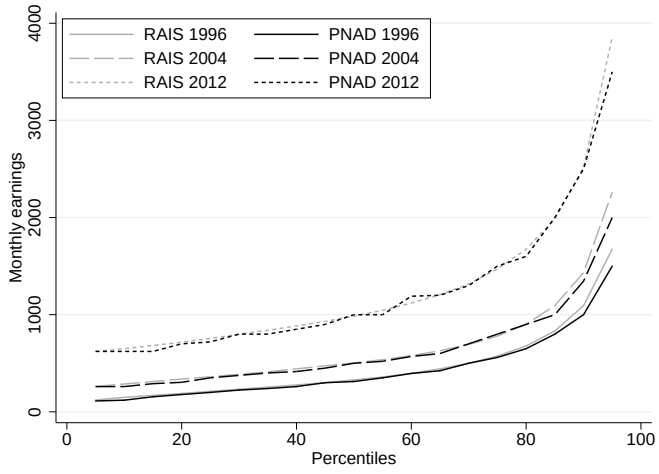
Online Appendix (Not for Publication)

Appendix A presents additional figures and tables. Appendix B calculates workers' and firms' formal sector wage valuations. Appendix C details the calculations using the Fréchet distribution. Appendix D extends the stylized model to monopolistic competition in the goods market. Appendix E details the proofs for all propositions in the main text. Appendix F calculates the goods market clearing. Appendix G details the computation of the quantitative model. Appendix H performs the Kaitz analysis of the effects of the minimum wage. Appendix I adds the unemployment margin to the quantitative model. Lastly, Appendix J conciliates my findings with those in Derenoncourt et al. (2021).

A Additional tables and figures

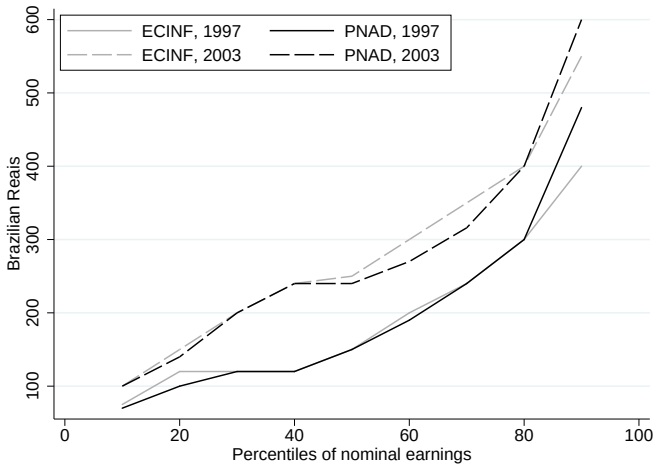
This Appendix contains additional figures and tables referenced in the main text.

Figure A.1: Comparison between RAIS and PNAD data sets, 1996-2012



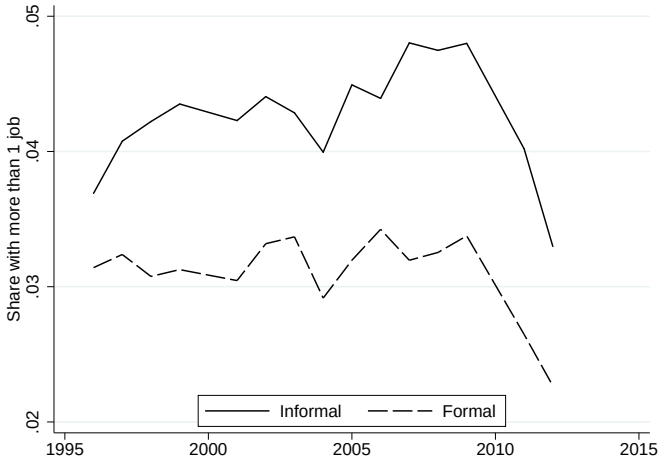
Notes: Comparison between formal earnings distributions in PNAD (black) and RAIS (grey) across different years (line patterns). *Sources:* 1996-2012 PNAD and RAIS.

Figure A.2: Comparison between ECINF and PNAD data sets, 1997 and 2003



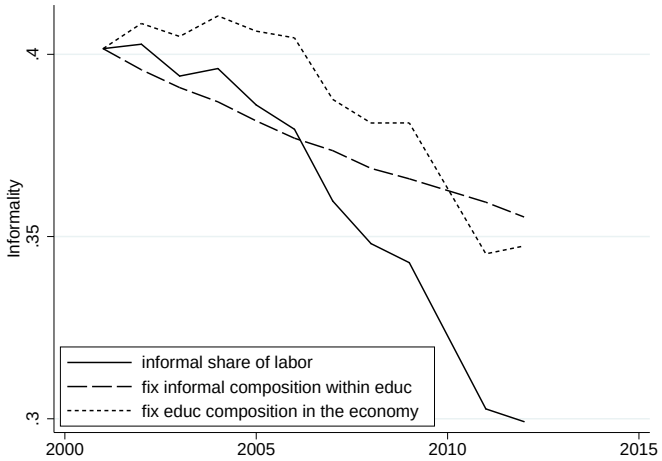
Notes: Comparison between informal earnings distributions in PNAD (black) and ECINF (grey) across different years (line patterns). *Sources:* 1997 and 2003 PNAD and ECINF.

Figure A.3: Share of formal/informal workers with more than one job, 1996-2012



Notes: Share of workers in the formal and informal sectors with more than one job in the reference week. *Sources:* 1997-2012 PNAD.

Figure A.4: Shift share decomposition of informality across education groups, 1996-2012



Notes: This figure performs a shift share decomposition of the informal share of labor across different education groups: $(L_t^I/L_t) = \sum_e (L_{et}/L_t) \cdot (L_{et}^I/L_{et})$ where e denotes education groups, t time and superscript I denotes informal employment. The solid line shows the observed movement in informal share of labor. The long dashes plot a counterfactual curve that fixes the share of informality within education groups (L_{et}^I/L_{et}) in its initial value. The short dashes plot a counterfactual curve that fixes the educational composition of the labor force (L_{et}/L_t) in its 1996 value. *Sources:* 1996-2012 PNAD.

Table A.1: Informal share in different industries

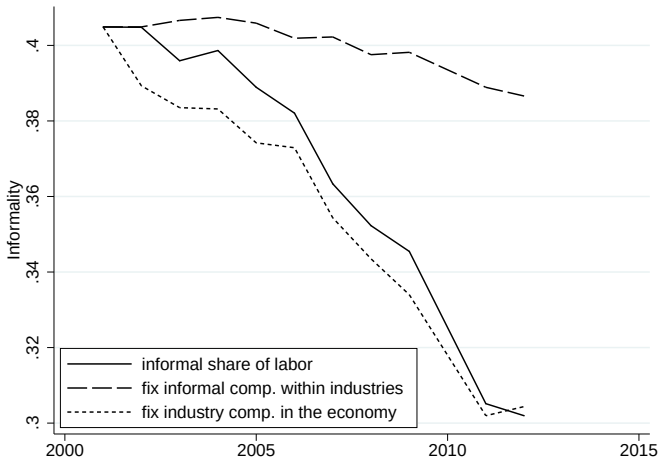
	Share informal	Share of total employment
Manufacturing	16.5	18.1
Other activities	16.5	9.8
Transport, storage, and communic.	20.1	5.8
Commerce and repair	24.5	18.2
Undefined	30.4	0.0
Education, health, and social serv.	32.8	9.5
Restaurant and accommodation	38.8	5.6
Construction	43.5	6.5
Other services	46.4	3.5
Public admin	55.2	3.5
Agriculture	61.6	7.8
Domestic services	69.4	11.7

Notes: Table restricts data to 2001-2012 period, as industry definitions are consistent across surveys. The second column shows the share of employment that is informal in each industry. The third column shows the size of each industry in terms of total employment. Sample weights are used. *Sources:* 2001/2012 PNAD.

Table A.2: Brazilian states and respective treatment groups

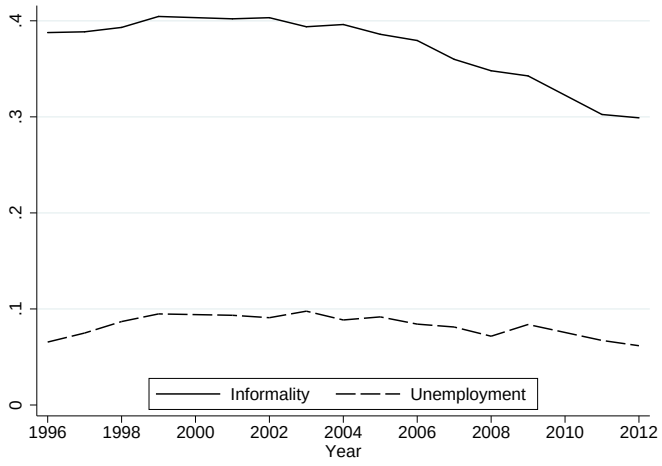
Group	State	Group	State
1	São Paulo	6	Pará
1	Santa Catarina	6	Paraíba
1	Distrito Federal	6	Acre
2	Amapá	7	Maranhão
2	Paraná	7	Pernambuco
2	Amazonas	7	Ceará
3	Mato Grosso	8	Alagoas
3	Rio de Janeiro	8	Tocantins
3	Rio Grande do Sul	8	Rio Grande do Norte
4	Rondônia	9	Bahia
4	Mato Grosso do Sul	9	Sergipe
4	Roraima	9	Piauí
5	Goiás		
5	Espírito Santo		
5	Minas Gerais		

Figure A.5: Shift share decomposition of informality across industries, 1996-2012



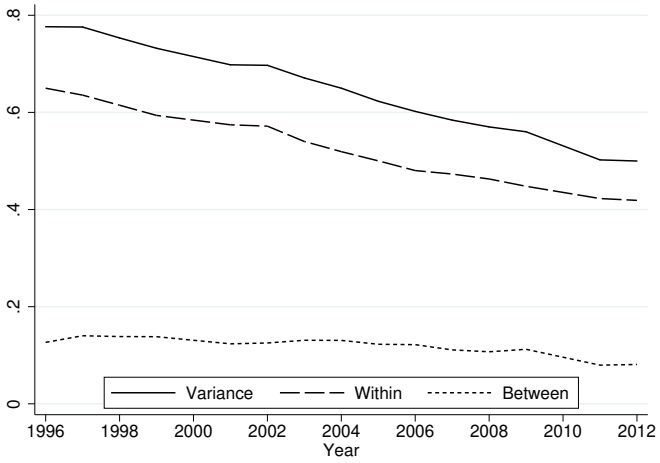
Notes: This figure performs a shift share decomposition of the informal share of labor across different industries: $(L_t^I/L_t) = \sum_j (L_{jt}/L_t) \cdot (L_{jt}^I/L_{jt})$ where j denotes industry, t time and superscript I denotes informal employment. The solid line shows the observed movement in informal share of labor. The long dashes plot a counterfactual curve that fixes the share of informality within industries (L_{jt}^I/L_{jt}) in its initial value. The short dashes plot a counterfactual curve that fixes the industry composition of the labor force (L_{jt}/L_t) in its 1996 value. *Sources:* 1996-2012 PNAD.

Figure A.6: Informality and unemployment, 1996-2012



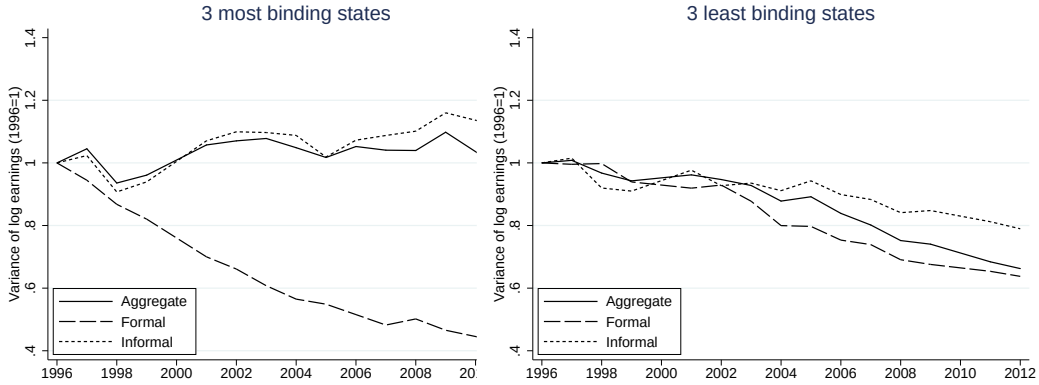
Notes: Solid line shows the fraction of informal workers. Long dasehs show the share of unemployed workers. *Sources:* 1996-2012 PNAD.

Figure A.7: Decomposition of overall variance of log earnings



Notes: This figure decomposes overall variance in log earnings into within and between terms, following Equation (1). *Sources:* 1996-2012 PNAD.

Figure A.8: Earnings inequality in states most and least exposed (incl. self employed)



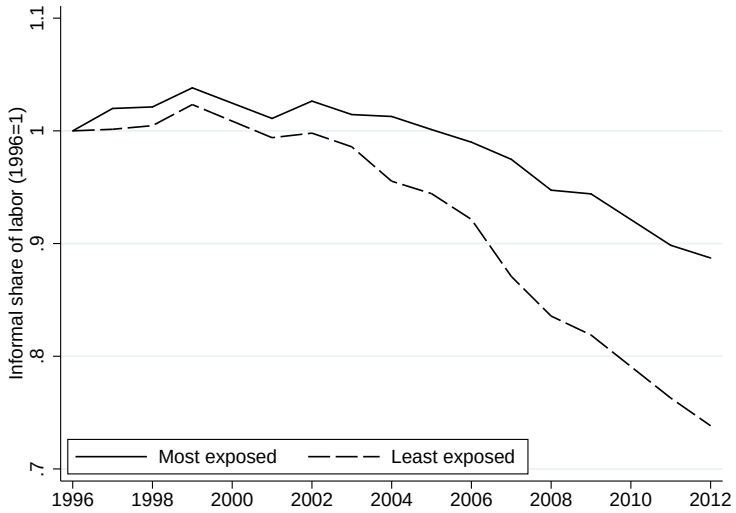
Notes: Evolution of earnings inequality (1996 normalized to 1) in the formal (long dashes) and informal (short dashes) sectors, and in the aggregate (solid lines). The plot on the left displays employment-weighted averages across the 3 states most binding (Piauí, Sergipe, and Bahia). The plot on the right displays employment-weighted averages across the 3 states least binding (São Paulo, Santa Catarina, and Distrito Federal). *Sources:* 1996-2012 PNAD.

Table A.3: Diff-in-diff results (complete table)

	$\log(V^{\text{All}})$	$\log(V^F)$	$\log(V^I)$	log(Inf Share)
β_2	-0.142	-0.203	-0.080	0.046
	(0.058)**	(0.046)***	(0.073)	(0.037)
β_3	0.010	-0.060	0.020	0.032
	(0.047)	(0.052)	(0.063)	(0.070)
β_4	0.083	-0.136	0.163	0.038
	(0.063)	(0.061)**	(0.107)	(0.067)
β_5	0.032	-0.103	0.117	-0.062
	(0.048)	(0.043)**	(0.064)*	(0.031)*
β_6	-0.008	-0.222	-0.020	0.013
	(0.063)	(0.091)**	(0.091)	(0.033)
β_7	0.085	-0.265	0.173	0.043
	(0.065)	(0.110)**	(0.075)**	(0.043)
β_8	0.213	-0.261	0.297	0.055
	(0.052)***	(0.075)***	(0.093)***	(0.021)**
β_9	0.200	-0.253	0.316	0.073
	(0.077)**	(0.063)***	(0.078)***	(0.032)**
Fraction high skill	0.485	0.447	0.582	-0.466
	(0.177)**	(0.373)	(0.254)**	(0.106)***
Fraction under 30	-0.561	-0.511	-0.742	0.219
	(0.165)***	(0.320)	(0.198)***	(0.143)
Fraction white	-0.083	-0.233	-0.200	0.052
	(0.165)	(0.204)	(0.172)	(0.064)
Fraction female	0.218	0.446	0.558	-0.059
	(0.147)	(0.269)	(0.220)**	(0.226)
Unemployment rate	-0.343	-1.689	-0.042	0.253
	(0.521)	(0.723)**	(0.668)	(0.336)
Observations	405	405	405	405
R^2	0.854	0.891	0.642	0.966

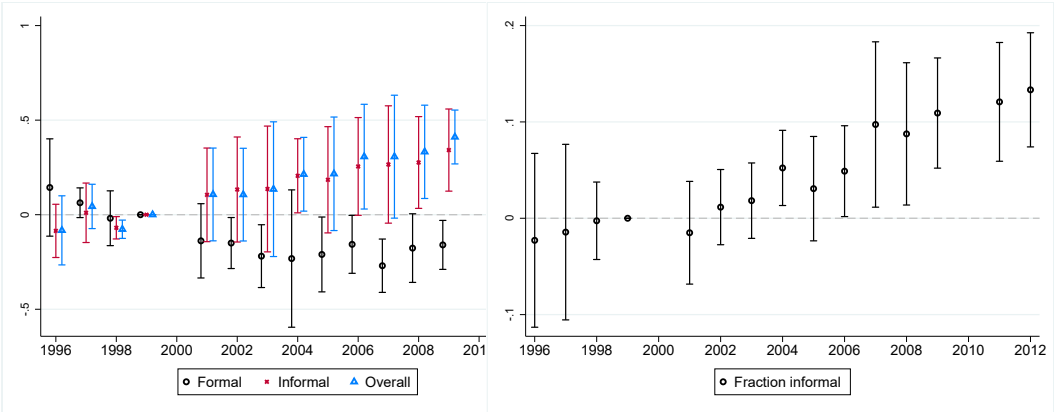
Notes: This table displays the coefficients of the OLS regression: $y_{sgt} = \alpha + \sum_{h \neq 1} \beta_h \cdot \mathcal{I}_{g=h} \cdot \mathcal{I}_{t>1999} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}$. Standard errors in parentheses are clustered at the state level. ***p<1%, **p<5%, *p<10%. *Sources:* 1996/2012 PNAD.

Figure A.9: Informal share in states most and least exposed (incl. self employed)



Notes: Evolution of the informal share of labor (1996 normalized to 1) in the most restricted states (solid line) versus least restricted states (long dashes). *Sources:* 1996-2012 PNAD.

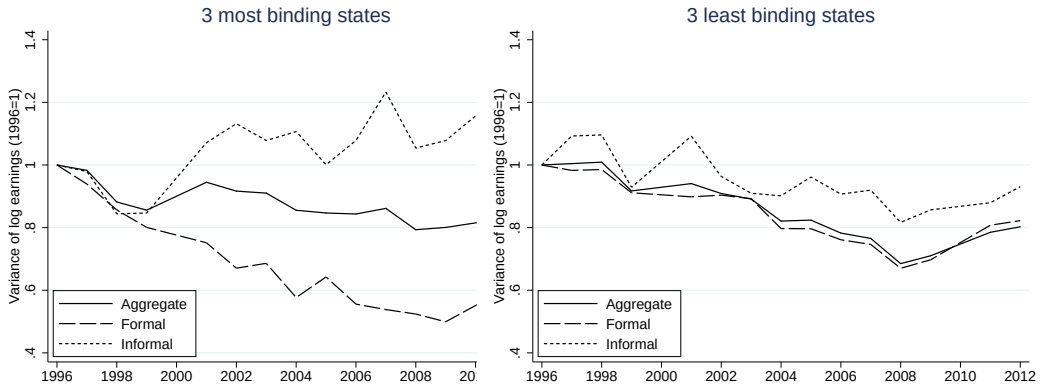
Figure A.10: Event study analysis (incl. self employed)



(a) log(Variance) (b) log(Informal share)

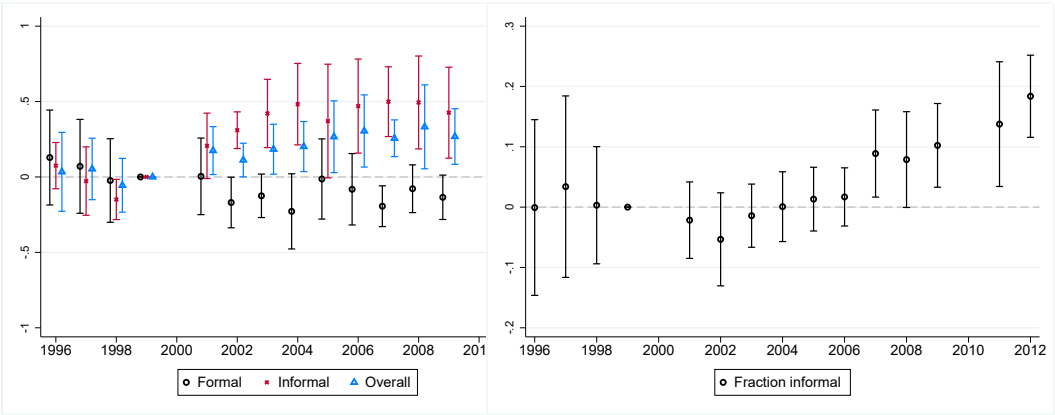
Notes: This figure plots the ordinary least squares coefficients of Equation (2) for the states in the most treated group, β_{k9} for $k \neq 1999$. Panel (a) displays the results for outcomes related to earnings inequality. Panel (b) displays the results for the log of the informa share. Standard errors are clustered at the state level. *Sources:* 1996-2012 PNAD.

Figure A.11: Earnings inequality in states most and least exposed (hourly earnings)



Notes: Evolution of earnings inequality (1996 normalized to 1) in the formal (long dashes) and informal (short dashes) sectors, and in the aggregate (solid line). The plot on the left displays employment-weighted averages across the 3 states most binding (Piauí, Sergipe, and Bahia). The plot on the right displays employment-weighted averages across the 3 states least binding (São Paulo, Santa Catarina, and Distrito Federal). *Sources:* 1996-2012 PNAD.

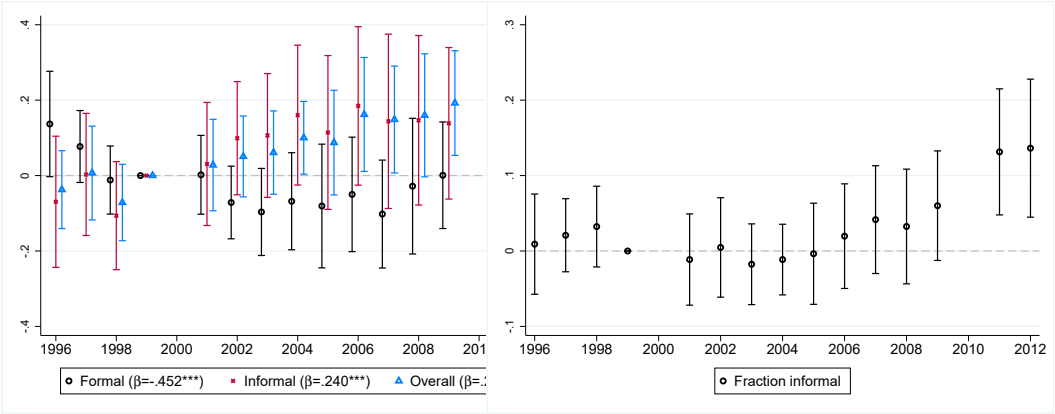
Figure A.12: Event study analysis (hourly earnings)



(a) log(Variance) (b) log(Informal share)

Notes: This figure plots the ordinary least squares coefficients of Equation (2) for the states in the most treated group, β_{k9} for $k \neq 1999$. Panel (a) displays the results for outcomes related to earnings inequality. Panel (b) displays the results for the log of the informal share. Standard errors are clustered at the state level. *Sources:* 1996-2012 PNAD.

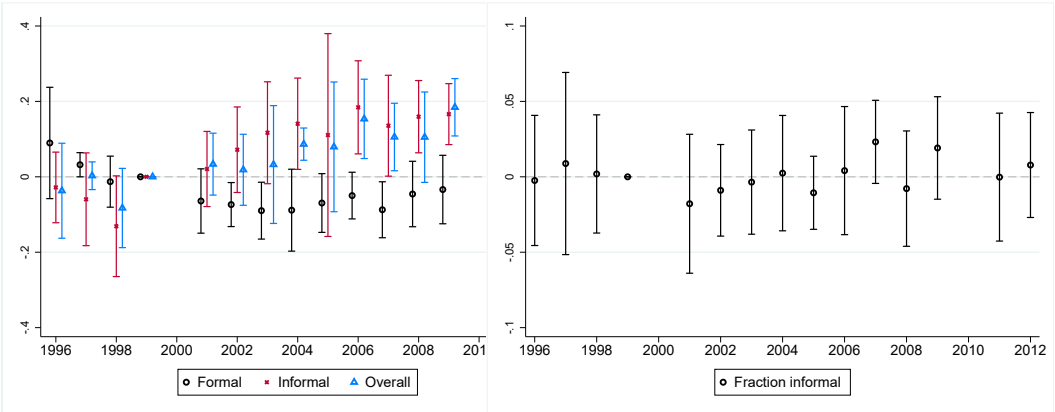
Figure A.13: Event study analysis (two groups, above/below median)



(a) log(Variance) (b) log(Informal share)

Notes: This figure plots the ordinary least squares coefficients for a variant of Equation (2) with only two groups - states above an below median 1999 share of formal minimum wage workers. Panel (a) displays the results for outcomes related to earnings inequality. Panel (b) displays the results for the log of the informal share. Standard errors are clustered at the state level. *Sources:* 1996-2012 PNAD.

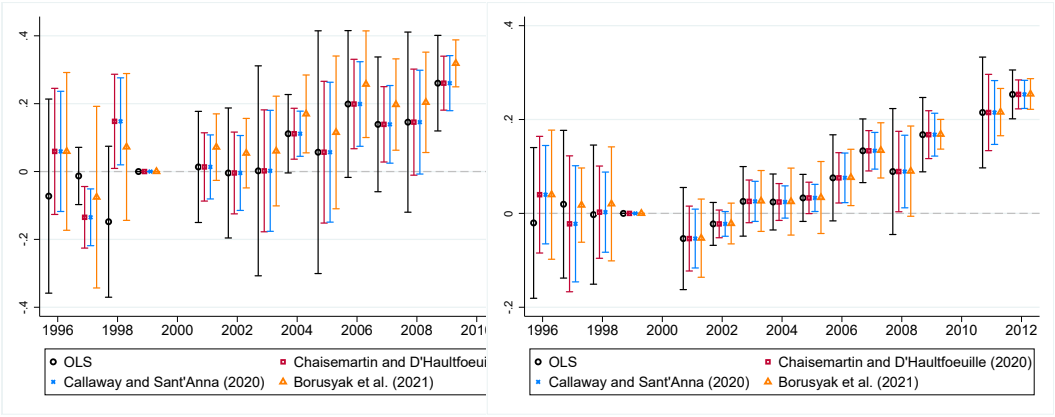
Figure A.14: Event study analysis (levels)



(a) Variance (b) Informal share

Notes: This figure plots the ordinary least squares coefficients of Equation (2) for the states in the most treated group, β_{k9} for $k \neq 1999$. Panel (a) displays the results for outcomes related to earnings inequality. Panel (b) displays the results for the informal share. Standard errors are clustered at the state level. *Sources:* 1996-2012 PNAD.

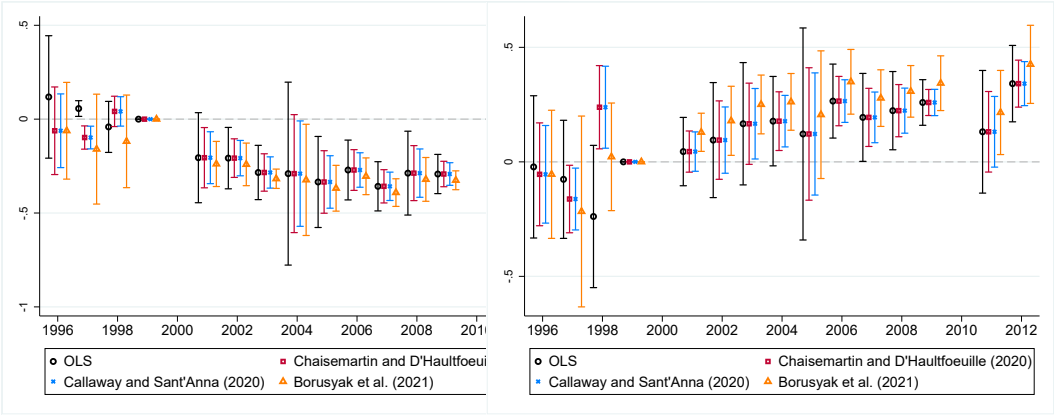
Figure A.15: Robustness to different TWFE estimators (1/2)



(a) log(Overall variance) (b) log(Informal share)

Notes: This figure plots different estimators of the two-way fixed effect model in Equation (2) for the states in the most treated group, β_{k9} for $k \neq 1999$. Standard errors are clustered at the state level. *Sources:* 1996-2012 PNAD.

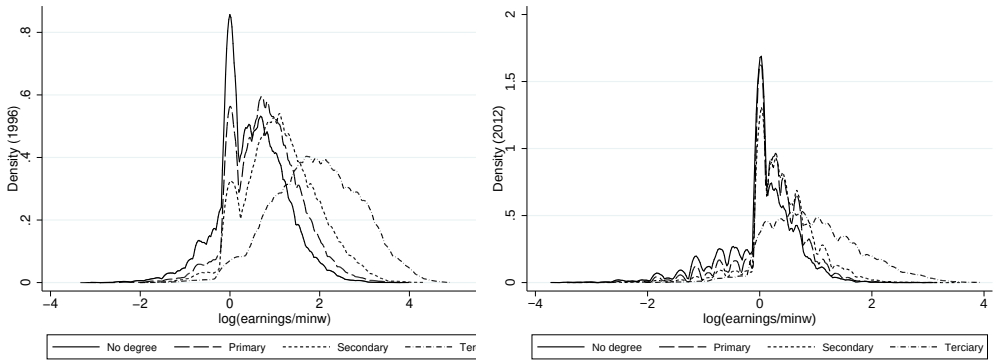
Figure A.16: Robustness to different TWFE estimators (2/2)



(a) log(Formal variance) (b) log(Informal variance)

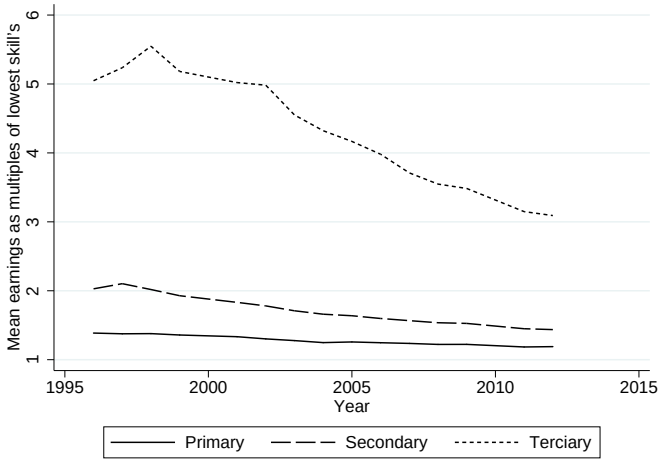
Notes: This figure plots different estimators of the two-way fixed effect model in Equation (2) for the states in the most treated group, β_{k9} for $k \neq 1999$. Standard errors are clustered at the state level. *Sources:* 1996-2012 PNAD.

Figure A.17: Earnings distribution relative to the minimum wage



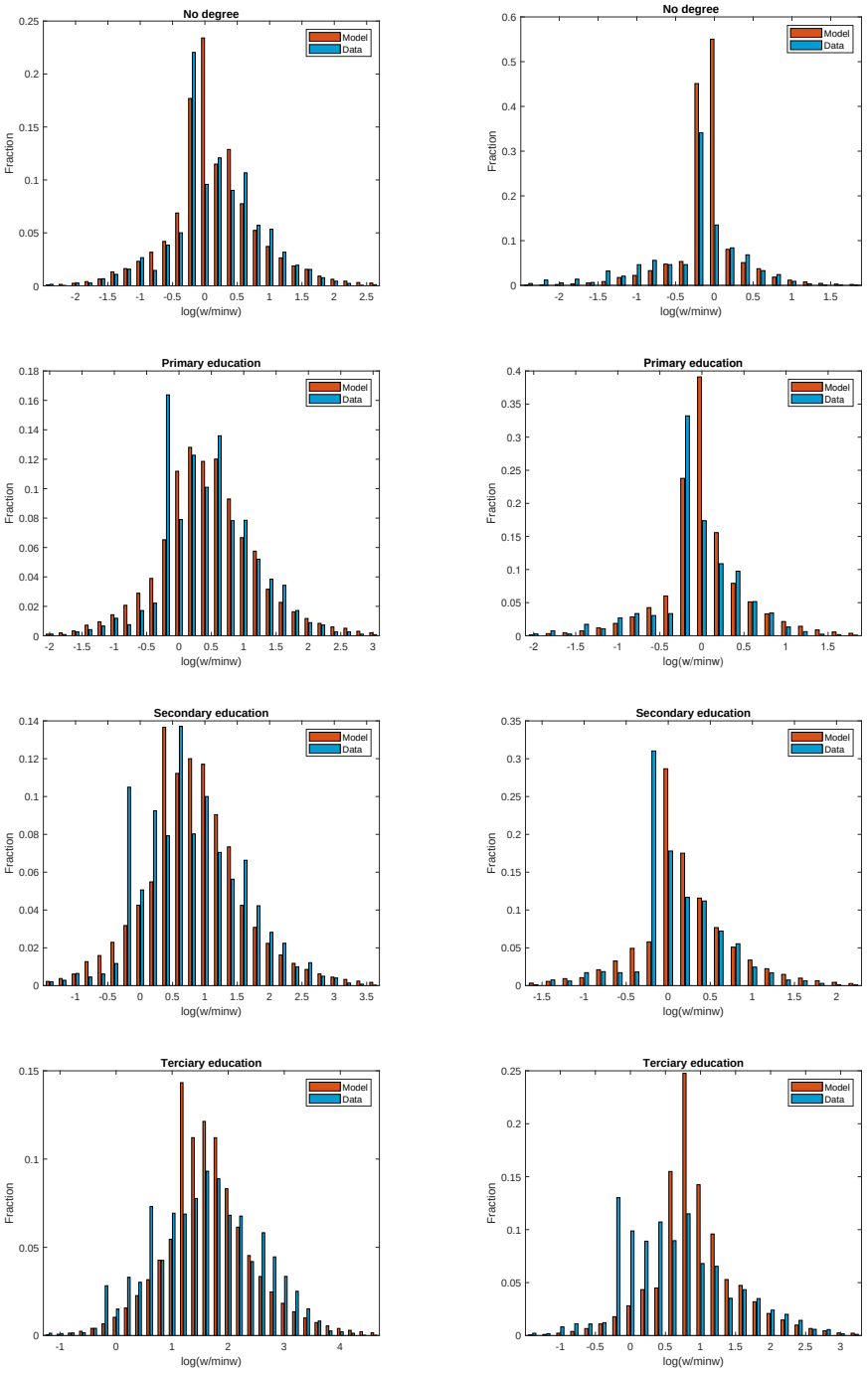
Notes: This figure plots different estimators of the two-way fixed effect model in Equation (2) for the states in the most treated group, β_{k9} for $k \neq 1999$. Standard errors are clustered at the state level. *Sources:* 1996-2012 PNAD.

Figure A.18: Mean earnings relative to non-degree mean earnings, 1996-2012



Notes: Mean earnings ratio between each educational category and non-degree workers. *Sources:* 1996-2012 PNAD.

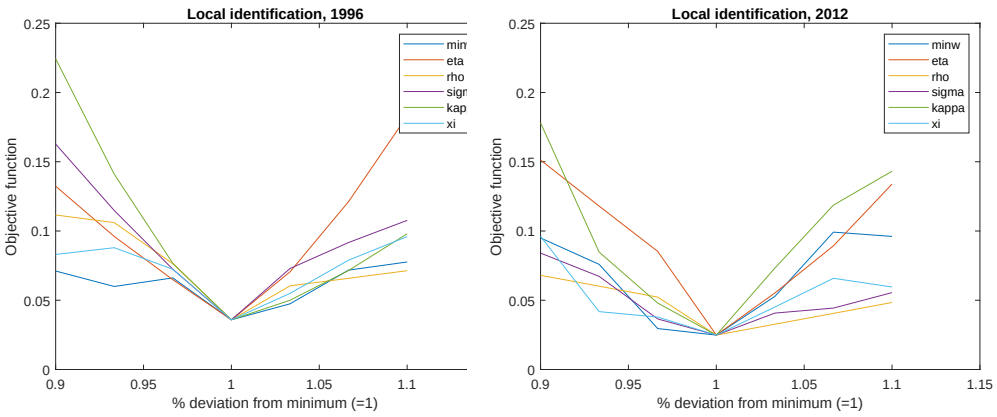
Figure A.19: Log earnings histogram by skills, 1996-2012



(a) 1996 (b) 2012

Notes: Histograms of log earnings relative to the minimum wage. Widths are set to 0.2. Blue histograms are model, red are data. *Sources:* 1996 and 2012 PNAD and model simulations.

Figure A.20: Identification of calibrated parameters



Notes: Each line plots the effect of a marginal change in a parameter value on the objective function ($= \sum_{i=1}^{8} |m_i(\Theta)/\hat{m}_i - 1|$). The parameters are centered around their calibrated values for 1996 and 2012. For brevity I vary all the ξ_h parameters at once, but the overall shapes in the figures do not change if I change one at a time. *Sources:* Model simulations.

B Calculation of workers' and firms' wage valuations

This appendix details the methodology used to estimate the valuation of formal nominal wages for workers and firms (ς_h and τ). I closely follow the work in Haanwinckel and Soares (Forthcoming) and Souza et al. (2012). The main idea is that households and firms value additional payments they receive, or have to incur, because of labor legislation.

I start by estimating the total labor cost of hiring a formal worker at a nominal monthly wage of 100 Brazilian Reais. The results are displayed in Table B.1. First, formal workers are entitled to a 13th salary by the end of the year (A.1). Second, the firm must pay a vacation stipend of 1/3 of the monthly wage (A.2). Third, in the period of 30 days prior to dismissal (Advance notice), formal employees can spend up to 25% of their work time searching for a new job. As discussed in Gonzaga et al. (2003), this advance notification is in practice an additional severance payment, as workers are not expected to put effort into working during that month.

The three items above represent transfers from firms to workers. I now discuss government taxation, which falls upon the raw total wage (B). In Brazil, formal workers have a severance payment fund (FGTS), where withdrawal can occur at the time of dismissal. Firms must make monthly contributions of 8% of the raw total wage to this fund (B.1). I estimate the total value of the FGTS fund by multiplying the monthly contribution times the average duration of a formal job from Haanwinckel and Soares (Forthcoming). Upon firing a worker, firms must incur severance payments of 50% of the value of the FGTS fund, with 40% going directly to the worker and 10% going to the government. Firms must also contribute to the retirement fund of the worker (INSS), as well as other social security contributions, which amount to a total of 25.3% of the raw total wage. Lastly, formal workers have one month of paid vacation per year. Hence, an adjustment factor of 1/11 is needed to represent the fact that the employee is only productive during 11 months in a year. These calculations result in an effective payroll tax rate of 71.4%.

To calculate valuation of formal nominal wages for workers, I apply the labor legislation to each observation in the PNAD data and average the resulting wedges across the educational groups. This process generates Figure 9. To illustrate the procedure, Table B.2 estimates the wedges in 1996 for four representative levels of earnings, corresponding to mean earnings in each education category in the data. The first three items are the direct transfers from firms to workers in terms of 13th

Table B.1: Calculating τ

Item	Formula	Value
Nominal wage (A)		100
13th salary (A.1)	A/12	8.33
Vacation (A.2)	(A/3)/12	2.78
Advance notice	(A+A.1+A.2)*dismiss prob.	3.33
Raw total (B)		114.44
FGTS contribution (B.1)	8% of B	9.16
FGTS fund (B.2)	B.1*duration	304.33
Severance payment	B.2/2*dismiss prob.	4.56
INSS employer	20% of B	22.89
Other contributions	5.3% of B	6.07
Total with contributions (C)		157.12
Vacation adjustment	C/11	14.28
Total cost (D)		171.40
Payroll tax rate: τ	D/A-1	71.4%

Notes: Calculation of payroll tax rate τ used in the model. The above calculations were made under a dismissal probability of 3% and expected duration of employment of 33 months (Haanwinckel, 2020). *Sources:* Labor legislation.

salary, vacation stipends and advance notices. Then comes the two largest deductions: worker contributions to the retirement system (INSS deduction) and income taxes. Importantly, these rates depend on the earnings level analyzed, a feature that is taken into account in these calculations. After that comes the valuation of the FGTS fund, severance payments made in the case of dismissal, and disability insurances. Lastly, one must adjust for the fact that workers are entitled to one month of paid vacations. The results show that the benefits accrued from having a formal labor contract more than compensate for the income and social security taxation (the wedges are positive). Moreover, notice that the wedges are larger for less-educated workers, a reflection of the progressiveness of the tax system in Brazil.

Table B.2: Calculating ς_h

Employee items	Formula	No degree	Primary	Secondary	Tertiary
Nominal wage (A)	Mean for each skill group	306	382	534	1323
13th salary (A.1)	A/12	25.5	31.8	44.5	110.3
Vacation (A.2)	(A/3)/12	8.5	10.6	14.8	36.8
Advance notice	(A+A.1+A.2)*dismiss prob.	10.2	12.7	17.8	44.1
Raw total (B)		350.2	437.2	611.1	1514.1
INSS deduction	INSS formula over (B)	-31.5	-39.3	-67.2	-105.3
Income tax	Income tax formula over (B)	0.0	0.0	0.0	-92.1
Valuation of FGTS	50% of firm contribution	14.0	17.5	24.4	60.6
Severance payment	40% FGTS fund*dismiss prob.	11.2	14.0	19.5	48.3
Accident insurance	2% of (B)	7.0	8.7	12.2	30.3
Total		350.9	438.0	600.1	1455.8
Vacation adjustment	Vacation cost paid by firm	43.7	54.6	76.3	189.0
Overall valuation		394.6	492.6	676.4	1644.8
Worker wedge		28.9%	28.9%	26.7%	24.3%

Notes: Calculation of formal worker wedges ς_h used in the model. The above calculations were made under a dismissal probability of 3% and expected duration of employment of 33 months (Haanwinckel, 2020). *Sources:* Labor legislation.

C Fréchet calculations

This Appendix details the Fréchet calculations of the labor supply curve at the firm level, as well as worker welfare, for the stylized model. Results for the quantitative model extend trivially, and are not derived for brevity.

Assume that the utility of household i working at firm j reads:

$$U_i(j) = A_i(j)w(j) \tag{28}$$

where we assume that the amenity shocks $A_i(j)$ are iid and follow a Fréchet distribution with shape η, scale equals to one and location equals to zero:

$$F(A(j)) = e^{-A(j)^{-\eta}}, \quad f(A(j)) = e^{-A(j)^{-\eta}}\eta A(j)^{-\eta-1} \tag{29}$$

The share of households that optimally choose firm j is:[21]

$$l(j) = \int_0^1 Pr\left(U_h(j) \geq U_h(j') \ \ \forall j' \neq j\right) dh \tag{30}$$

$$l(j) = \int_0^1 \int_0^\infty f(A(j)) \prod_{j' \in \Omega \setminus \{j\}} F\left(\frac{w(j)A(j)}{w(j')}\right) dA(j)dh \tag{31}$$

$$l(j) = \int_0^1 \int_0^\infty e^{-A(j)^{-\eta}}\eta A(j)^{-\eta-1} \prod_{j' \in \Omega \setminus \{j\}} e^{-\left(\frac{w(j)A(j)}{w(j')}\right)^{-\eta}} dA(j)dh \tag{32}$$

$$l(j) = \int_0^1 \int_0^\infty \eta A(j)^{-\eta-1} e^{-\int_{j' \in \Omega}\left(\frac{w(j)A(j)}{w(j')}\right)^{-\eta}} dA(j)dh \tag{33}$$

$$l(j) = \int_0^1 \int_0^\infty \eta A(j)^{-\eta-1} e^{-(w(j)A(j))^{-\eta}\int_{j' \in \Omega} w(j')^\eta} dA(j)dh \tag{34}$$

$$l(j) = \int_0^1 \int_0^\infty \eta A(j)^{-\eta-1} e^{-\left(\frac{A(j)}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}\right)^{-\eta}} dA(j)dh \tag{35}$$

[21] In the case of a discrete number of firms $j = 1, ..., J$, the labor share allocated at firm 1 would be:

$$l(1) = \int_0^1 \int_0^\infty \int_0^{\frac{w(1)A(1)}{w(2)}} ... \int_0^{\frac{w(1)A(1)}{w(J)}} f(A(J))...f(A(1))\, dA(J)...dA(2)dA(1)dh$$

which denotes the probability that firm 1 is chosen over all other firms $j = 2, ..., J$ in the economy. The equation for the continuum of firms is an alternative form of expressing the same variable, calculated in Desmet et al. (2018).

Define $s \equiv \frac{\left[\int_{j' \in \Omega} w(j')^{\eta}\right]^{1/\eta}}{w(j)}$, and manipulate to find:

$$l(j) = s^{-\eta} \int_0^1 \int_0^{\infty} \frac{\eta}{s} \left[\frac{A(j)}{s}\right]^{-\eta-1} e^{-\left(\frac{A(j)}{s}\right)^{-\eta}} dA(j) dh \tag{36}$$

and use the fact that households are homogeneous (so integral over h is irrelevant), and that $\frac{\eta}{s}\left[\frac{A(j)}{s}\right]^{-\eta-1} e^{-\left(\frac{A(j)}{s}\right)^{-\eta}}$ is the pdf of a Fréchet distribution with shape η and scale s (so it integrates to one) to find:

$$l(j) = \frac{w(j)^{\eta}}{\int_{j' \in \Omega} w(j')^{\eta}} \tag{37}$$

I now calculate what is the expected utility of a household in the model. The probability that the utility of household i being less than u conditional on firm j being its optimal choice is:

$$F_U(u) = Pr(U_i(j) \leq u \,|\, U_i(j) \geq U_i(j') \,\forall j' \neq j) = \frac{Pr(U_i(j) \leq u \;\&\; U_i(j) \geq U_i(j') \,\forall j' \neq j)}{Pr(U_i(j) \geq U_i(j') \,\forall j' \neq j)} \tag{38}$$

$$F_U(u) = \frac{\int_0^{\frac{u}{w(j)}} f(A(j)) \prod_{j' \in \Omega \setminus \{j\}} F\left(\frac{w(j)A(j)}{w(j')}\right) dA(j)}{\int_0^{\infty} f(A(j)) \prod_{j' \in \Omega \setminus \{j\}} F\left(\frac{w(j)A(j)}{w(j')}\right) dA(j)} \tag{39}$$

$$F_U(u) = \frac{\int_0^{\frac{u}{w(j)}} \eta A(j)^{-\eta-1} e^{-\left(\frac{\frac{A(j)}{\left[\int_{j' \in \Omega} w(j')^{\eta}\right]^{1/\eta}}}{\frac{w(j)}{}}\right)^{-\eta}} dA(j)}{\frac{w(j)^{\eta}}{\int_{j' \in \Omega} w(j')^{\eta}}} \tag{40}$$

$$F_U(u) = \int_0^{\frac{u}{w(j)}} \frac{\int_{j' \in \Omega} w(j')^{\eta}}{w(j)^{\eta}} \eta A(j)^{-\eta-1} e^{-\left(\frac{\frac{A(j)}{\left[\int_{j' \in \Omega} w(j')^{\eta}\right]^{1/\eta}}}{\frac{w(j)}{}}\right)^{-\eta}} dA(j) \tag{41}$$

$$F_U(u) = \int_0^{\frac{u}{w(j)}} \frac{\eta}{\left[\int_{j' \in \Omega} w(j')^{\eta}\right]^{1/\eta}} \left[\frac{w(j)A(j)}{\left[\int_{j' \in \Omega} w(j')^{\eta}\right]^{1/\eta}}\right]^{-\eta-1} e^{-\left(\frac{w(j)A(j)}{\left[\int_{j' \in \Omega} w(j')^{\eta}\right]^{1/\eta}}\right)^{-\eta}} w(j) dA(j) \tag{42}$$

change variables and call $x = w(j)A(j)$ to find:

$$F_U(u) = \int_0^u \frac{\eta}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}} \left[\frac{x}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}\right]^{-\eta-1} e^{-\left(\frac{x}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}\right)^{-\eta}} dx \tag{43}$$

so the optimal utility U is a Fréchet random variable with shape η and scale $\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}$, which means that its mean is given by:

$$\mathbb{E}[U] = \Gamma\left(\frac{\eta - 1}{\eta}\right)\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta} = \Gamma\left(\frac{\eta - 1}{\eta}\right) W \tag{44}$$

D Monopolistic competition

This section considers the case in which firms not only have monopsony power in the labor market but also are monopolistic competitors in the goods market. I assume each firm produces a different variety, which is demanded by workers in a CES fashion. I show that this changes slightly the problem of the firm, but does not alter the threshold characterization of the solution, in which low-productivity firms select into the informal sector. Hence the qualitative results in Section 2 do not change.

I first analyze the problem of the household. The consumption problem of household i working for firm j, consuming varieties from all other firms k is:

$$V_i(j) = \max_{c(k)} \left\{ A_i(j) \left[\int_{k\in\Omega} c_i(k)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad | \quad \int_{k\in\Omega} p(k)c_i(k) = w(j) \right\} \tag{45}$$

where σ is the elasticity of substitution between varieties. The first order conditions of this problem give rise to a downward sloping demand curve for the product of firm k consumed by household i employed at firm j:

$$c_i(k) = \left[\frac{p(k)}{P} \right]^{-\sigma} \frac{w(j)}{P} \tag{46}$$

where $P = \left[\int_{k\in\Omega} p(k)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$ denotes the CES price index such that $PC_i(j) = w(j)$ with $C_i(j) = \left[\int_{k\in\Omega} c_i(k)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$.

The solution for the consumption problem yields the following indirect utility function:

$$V_i(j) = \frac{A_i(j)w(j)}{P} \tag{47}$$

and the employment decision of household i boils down to selecting the employer that offers the highest amenity-adjusted wage:

$$U_i = \max_{j\in\Omega} \left\{ \frac{A_i(j)w(j)}{P} \right\} \tag{48}$$

First, notice that the introduction of monopolistic competition in the goods market does not alter the choice probabilities of workers to different firms, hence

the labor supply curve faced by firm j still reads:

$$l(j) = \left(\frac{w(j)}{W}\right)^{\eta} \tag{49}$$

where $W = \left[\int_{j\in\Omega} w(j)^{\eta}\right]^{\frac{1}{\eta}}$ denotes the aggregate wage index. Second, using calculations similar to those in Appendix C, it is easy to calculate the aggregate demand for products from firm $k \in \Omega$, coming from all households working at all firms $j \in \Omega$:

$$c(k) = \left[\frac{p(k)}{P}\right]^{-\sigma} \frac{W}{P} \tag{50}$$

This result comes from the fact that the aggregate wage index represents the total amount of earnings earned by households after their optimal employment decisions. So far I have distinguished the monopsonist j from the monopolist k, but from now on I will look at the labor supply and product demand curves for the same firm.

Consider the problem of the informal firm operating with productivity z:

$$\pi^{inf}(z) = \max_{\{c,p,l,w\}} \left\{(1-\rho)pc - wl \quad | \quad c = zl, \;\; c = P^{\sigma-1}W \cdot p^{-\sigma}, \;\; l = W^{-\eta} \cdot w^{\eta}\right\} \tag{51}$$

The first constraint is the linear production function, the second constraint represents market power in the goods market, and the last constraint represents the monopsony power.

Substitute the constraints in this problem to find:

$$\pi^{inf}(z) = \max_{\{l\}} \left\{(1-\rho)P^{\frac{\sigma-1}{\sigma}}W^{\frac{1}{\sigma}}(zl)^{1-\frac{1}{\sigma}} - Wl^{1+\frac{1}{\eta}}\right\} \tag{52}$$

and the solution reads:

$$l^{inf}(z) = (W/P)^{-\frac{\sigma-1}{\sigma}\frac{\eta\sigma}{\eta+\sigma}} \left(\frac{\sigma-1}{\sigma}\frac{\eta}{\eta+1}\right)^{\frac{\eta\sigma}{\eta+\sigma}} (1-\rho)^{\frac{\eta\sigma}{\eta+\sigma}} z^{\frac{\sigma-1}{\sigma}\frac{\eta\sigma}{\eta+\sigma}} \tag{53}$$

Importantly, define the adjusted markdown as $\tilde{\eta} \equiv \frac{\sigma-1}{\sigma}\frac{\eta\sigma}{\eta+\sigma}$ and the real wage of workers $\tilde{W} \equiv W/P$ to find:

$$l^{inf}(z) = \tilde{W}^{-\tilde{\eta}} \left(\frac{\tilde{\eta}}{\tilde{\eta}+1}\right)^{\frac{\sigma}{\sigma-1}\tilde{\eta}} (1-\rho)^{\frac{\sigma}{\sigma-1}\tilde{\eta}} z^{\tilde{\eta}} \tag{54}$$

There are two takeaways from the above equation. First, as σ goes to infinity we have varieties that are perfect substitutes, and we get back the same expression as in the main text. Second, the labor allocation, and consequently profits in the informal sector are a "modified" version of the ones derived in the main text, except that now there is curvature with respect to productivity z that comes from both the elasticity of the labor supply curve, η, and the elasticity of the demand curve, σ.

The intuitions and results for the firms in the formal sector are available upon request.

E Theory appendix

This section details the proofs for all propositions in the main text.

E.1 Proof of Proposition 1

Firms with $z \leq \underline{w}$ experience negative profits in the formal sector, hence operate the informal technology. On the other hand, when $z \geq \bar{z} \equiv \frac{\eta+1}{\eta}\underline{w}$ minimum wages do not bind, and profits in the formal sector are greater than profits in the informal sector because of the informal sector cost.

I analyze firms with $z \in (\underline{w}, \bar{z})$, where minimum wage is binding but there are positive profits in the formal sector. Define the profit gains from operating informally as:

$$\Delta(z) \equiv W^{-\eta}\left[\frac{\eta^\eta}{(\eta+1)^{\eta+1}}(1-\rho)^{\eta+1}z^{\eta+1} - \underline{w}^\eta z + \underline{w}^{\eta+1}\right] \tag{55}$$

It is easy to see that $\Delta(\underline{w}) > 0$ and $\Delta(\bar{z}) < 0$, as well as $\Delta^{'}(z) < 0$ for $z \in (\underline{w}, \bar{z})$. Hence, there exists a threshold $\underline{z} \in (\underline{w}, \bar{z})$ implicitly determined by:

$$\frac{\eta^\eta}{(\eta+1)^{\eta+1}}(1-\rho)^{\eta+1}\underline{z}^{\eta+1} - \underline{w}^\eta \underline{z} + \underline{w}^{\eta+1} = 0 \tag{56}$$

where firms with productivity below $\underline{z}$ operate informally, and firms with productivity above $\underline{z}$ operate the formal technology restricted by the minimum. The ordering $\underline{w} \leq \underline{z} < \bar{z}$ follows from the above arguments.

For the third part, differentiate Equation (56) to find:

$$\frac{\partial \underline{z}}{\partial \rho} = \frac{\frac{\eta^\eta}{(\eta+1)^\eta}(1-\rho)^\eta \underline{z}^{\eta+1}}{\frac{\eta^\eta}{(\eta+1)^\eta}(1-\rho)^{\eta+1}\underline{z}^\eta - \underline{w}^\eta} < 0, \quad \frac{\partial \underline{z}}{\partial \underline{w}} = \frac{\eta \underline{w}^{\eta-1}\underline{z} - (\eta+1)\underline{w}^\eta}{\frac{\eta^\eta}{(\eta+1)^\eta}(1-\rho)^{\eta+1}\underline{z}^\eta - \underline{w}^\eta} > 0 \tag{57}$$

$$\frac{\partial^2 \underline{z}}{\partial \rho \partial \underline{w}} = \frac{\eta \underline{w}^{\eta-1}\frac{\partial \underline{z}}{\partial \rho} + \frac{\eta^\eta}{(\eta+1)^{\eta-1}}(1-\rho)^\eta \underline{z}^\eta \frac{\partial \underline{z}}{\partial \underline{w}} - \frac{\eta^{\eta+1}}{(\eta+1)^\eta}(1-\rho)^{\eta+1}\underline{z}^{\eta-1}\frac{\partial \underline{z}}{\partial \rho}\frac{\partial \underline{z}}{\partial \underline{w}}}{\frac{\eta^\eta}{(\eta+1)^\eta}(1-\rho)^{\eta+1}\underline{z}^\eta - \underline{w}^\eta} < 0 \tag{58}$$

To show the fourth part, divide the cutoff equation by $\underline{w}^{\eta+1}$ to find:

$$\frac{\eta^\eta}{(\eta+1)^{\eta+1}}(1-\rho)^{\eta+1}(\underline{z}/\underline{w})^{\eta+1} - \underline{z}/\underline{w} + 1 = 0 \tag{59}$$

hence, the ratio between the informality threshold and the minimum wage is entirely determined by η and ρ.

E.2 Proof of Proposition 3

Differentiate aggregate labor demand with respect to the minimum wage:

$$\frac{\partial L^D}{\partial \underline{w}} = l^{inf}(\underline{z})f(\underline{z})\frac{\partial \underline{z}}{\partial \underline{w}} + \left[f(\bar{z})\frac{\partial \bar{z}}{\partial \underline{w}} - f(\underline{z})\frac{\partial \underline{z}}{\partial \underline{w}}\right] l^{\underline{w}} + [F(\bar{z}) - F(\underline{z})]\frac{\partial l^{\underline{w}}}{\partial \underline{w}} - l^{form}(\bar{z})f(\bar{z})\frac{\partial \bar{z}}{\partial \underline{w}} \tag{60}$$

but $l^{form}(\bar{z}) = l^{\underline{w}}$, so the above becomes:

$$\frac{\partial L^D}{\partial \underline{w}} = [F(\bar{z}) - F(\underline{z})]\frac{\partial l^{\underline{w}}}{\partial \underline{w}} - \left[l^{\underline{w}} - l^{inf}(\underline{z})\right] f(\underline{z})\frac{\partial \underline{z}}{\partial \underline{w}} \tag{61}$$

which shows the first part of the proposition using the fact that $\bar{z} > \underline{z}$ and $l^{\underline{w}} > l^{inf}(\underline{z})$.

To show the second part of the proposition, I will show that in the Pareto case the wage index decreases when minimum wages goes up. Given the inelastic labor supply curve, it then implies that labor demand must have decreased, which proves the result.

To simplify calculations, I study the behavior of $\tilde{W} \equiv \left(\frac{\eta+1}{\eta}W\right)^{\eta}$ as the minimum wage changes. When F is Pareto(z_0,$\nu > \eta$), $\tilde{W}$ takes the form:

$$\tilde{W} = (1-\rho)^{\eta}\frac{\nu}{\nu-\eta}\left(z_0^{-(\nu-\eta)} - \underline{z}^{-(\nu-\eta)}\right) + (\underline{z}^{-\nu} - \bar{z}^{-\nu})\bar{z}^{\eta} + \frac{\nu}{\nu-\eta}\bar{z}^{-(\nu-\eta)} \tag{62}$$

which can be rearranged as:

$$\tilde{W} = (1-\rho)^{\eta}\frac{\nu}{\nu-\eta}z_0^{-(\nu-\eta)} + \left\{\frac{\eta}{\nu-\eta} + \left[1 - \frac{\nu}{\nu-\eta}(1-\rho)^{\eta}(\underline{z}/\bar{z})^{\eta}\right](\underline{z}/\bar{z})^{-\nu}\right\}\bar{z}^{-(\nu-\eta)} \tag{63}$$

Hence, because $(\underline{z}/\bar{z})$ does not vary with the minimum wage (Proposition 1), and $1 - \frac{\nu}{\nu-\eta}(1-\rho)^{\eta}(\underline{z}/\bar{z})^{\eta} > 0$, increasing the minimum wage reduces the aggregate wage index.

E.3 Proof of Proposition 2

Throughout this proof I use a superscript I to denote workers in the informal sector, a superscript F to denote workers in the formal sector (both at and above the minimum wage), a superscript $\underline{w}$ to denote minimum wage workers and FNB to denote formal workers above the minimum.

I start with the following variance decomposition:

$$V = L^I V^I + L^F V^F + L^I \left(\mathbb{E}^I - \mathbb{E}\right)^2 + L^F \left(\mathbb{E}^F - \mathbb{E}\right)^2 \tag{64}$$

that decomposes overall variance of log earnings into the within and between components, splitting workers between the formal and informal groups.

Notice that aggregate variance of log earnings is: $\mathbb{E} = L^I \mathbb{E}^I + L^F \mathbb{E}^F$. Hence, I have:

$$V = L^I V^I + (1 - L^I) V^F + L^I (1 - L^I)(\mathbb{E}^I - \mathbb{E}^F)^2 \tag{65}$$

which I differentiate with respect to the minimum wage, to find:

$$\frac{\partial V}{\partial \underline{w}} = \frac{\partial L^I}{\partial \underline{w}} V^I + L^I \frac{\partial V^I}{\partial \underline{w}} - \frac{\partial L^I}{\partial \underline{w}} V^F + (1 - L^I) \frac{\partial V^F}{\partial \underline{w}} + \left(\frac{\partial L^I}{\partial \underline{w}} - 2 L^I \frac{\partial L^I}{\partial \underline{w}} \right) (..)^2 + L^I (1 - L^I) \frac{\partial (..)^2}{\partial \underline{w}} \tag{66}$$

where (..) denote the difference in mean log earnings in the formal and informal sectors.

Define $\underline{w}_0$ such that $\underline{z} = z_0$:

$$\frac{\eta^\eta}{(\eta + 1)^{\eta+1}} (1 - \rho)^{\eta+1} z_0^{\eta+1} - \underline{w}_0^\eta z_0 + \underline{w}_0^{\eta+1} = 0 \tag{67}$$

Importantly, at $\underline{w} \approx \underline{w}_0$, $L^I \approx 0$, implying:

$$\frac{\partial V}{\partial \underline{w}} = \frac{\partial V^F}{\partial \underline{w}} + \frac{\partial L^I}{\partial \underline{w}} \left[(\mathbb{E}^I - \mathbb{E}^F)^2 + V^I - V^F \right] \tag{68}$$

and establishing the first part of the proposition.

I now turn to the Pareto case. To show part 1. of the second half of the proposition, I calculate the variance of log earnings in an economy without the informal sector. I then show that increasing the minimum wage reduces inequality necessarily.

Importantly, we are in the case where $\underline{w}$ is such that $\underline{z} = z_0$, which implies that the minimum wage, though binding ($\bar{z} > z_0$), does not cause firm exit ($\underline{w} < z_0$). When $\underline{w} < z_0$, the share of the workforce at the minimum wage is:

$$L^{\underline{w}} = \frac{\frac{\nu - \eta}{\nu} \left[(w_0^F / \underline{w})^{-\nu} - 1 \right]}{\frac{\nu - \eta}{\nu} \left[(w_0^F / \underline{w})^{-\nu} - 1 \right] + 1}, \quad w_0^F \equiv \frac{\eta}{\eta + 1} z_0, \tag{69}$$

and the share of workers above the minimum wage is $L^{FNB} = 1 - L^{\underline{w}}$. By inspection, it is easy to see that when $\underline{w}$ increases, $L^{\underline{w}}$ goes up and L^{FNB} decreases.

The wage distribution (and corresponding density) takes the form:

$$G(w) = \begin{cases} 0 & w < \underline{w} \\ L^{\underline{w}} & w = \underline{w} \\ L^{\underline{w}} + (1 - L^{\underline{w}}) \cdot [1 - (\underline{w}/w)^{\nu-\eta}] & w > \underline{w} \end{cases} \tag{70}$$

$$g(w)dw = \begin{cases} 0 & w < \underline{w} \\ L^{\underline{w}} & w = \underline{w} \\ (1 - L^{\underline{w}})(\nu - \eta)\underline{w}^{\nu-\eta} w^{-(\nu-\eta+1)} dw & w > \underline{w} \end{cases} \tag{71}$$

With mean of log earnings:

$$\mathbb{E}[\log w] = \int_{\underline{w}}^{\infty} \log w g(w) dw + \log \underline{w} L^{\underline{w}} = \frac{L^{FNB}}{\nu - \eta} + \log \underline{w}, \tag{72}$$

and mean of square log earnings:

$$\mathbb{E}^F[(\log w)^2] = 2\frac{L^{FNB}}{\nu - \eta}\left[\log \underline{w} + \frac{1}{\nu - \eta}\right] + (\log \underline{w})^2 \tag{73}$$

so the variance is:

$$V(\log w) = \frac{2L^{FNB} - (L^{FNB})^2}{(\nu - \eta)^2}. \tag{74}$$

Notice that $2x - x^2$ is a concave parabola with maximum at $x = 1$. Hence, because we have $L^{FNB} \leq 1$, $\uparrow \underline{w} \Rightarrow \downarrow L^{FNB} \Rightarrow \downarrow V$.

I now show part 2. of the second half of the proposition. It states that when the informal sector is present, the formal sector response is zero and the informal sector response is positive, hence variance of aggregate log earnings increases with the minimum wage, at the margin.

I proceed in three steps. First, I show that when the informal sector is present, the relative share of minimum wage workers within the formal sector workers does not change with the minimum wage, and this implies that the variance of log earnings in the formal sector does not change with $\underline{w}$ ($FR = 0$). I then show that the share of informal workers strictly increase with the minimum wage, so $\frac{\partial L^I}{\partial \underline{w}} > 0$. I then prove that the last term, inside brackets, is also positive, implying that $IR > 0$ and

hence $\frac{\partial V}{\partial \underline{w}} > 0$.

In a model with Pareto distribution and informality, the share of informal and minimum wage workers are, respectively:

$$L^I = \frac{(w_0^I)^{-(\nu-\eta)} - (w_1^I)^{-(\nu-\eta)}}{(w_0^I)^{-(\nu-\eta)} - (w_1^I)^{-(\nu-\eta)} + \frac{\nu-\eta}{\nu}\left[(w_1^I)^{-\nu} - (1-\rho)^{-\nu}\underline{w}^{-\nu}\right]\underline{w}^{\eta} + (1-\rho)^{-\nu}\underline{w}^{-(\nu-\eta)}} \tag{75}$$

$$L^{\underline{w}} = \frac{\frac{\nu-\eta}{\nu}\left[(w_1^I)^{-\nu} - (1-\rho)^{-\nu}\underline{w}^{-\nu}\right]\underline{w}^{\eta}}{(w_0^I)^{-(\nu-\eta)} - (w_1^I)^{-(\nu-\eta)} + \frac{\nu-\eta}{\nu}\left[(w_1^I)^{-\nu} - (1-\rho)^{-\nu}\underline{w}^{-\nu}\right]\underline{w}^{\eta} + (1-\rho)^{-\nu}\underline{w}^{-(\nu-\eta)}} \tag{76}$$

Importantly, the relative share of formal sector workers at the minimum wage does not vary with the minimum wage:

$$\frac{L^{\underline{w}}}{1-L^I} = \frac{\frac{\nu-\eta}{\nu}\left[(w_1^I)^{-\nu} - (1-\rho)^{-\nu}\underline{w}^{-\nu}\right]\underline{w}^{\eta}}{\frac{\nu-\eta}{\nu}\left[(w_1^I)^{-\nu} - (1-\rho)^{-\nu}\underline{w}^{-\nu}\right]\underline{w}^{\eta} + (1-\rho)^{-\nu}\underline{w}^{-(\nu-\eta)}} = \frac{\frac{\nu-\eta}{\nu}\left[\left(\frac{\eta}{\eta+1}\underline{z}/\underline{w}\right)^{-\nu} - 1\right]}{\frac{\nu-\eta}{\nu}\left[\left(\frac{\eta}{\eta+1}\underline{z}/\underline{w}\right)^{-\nu} - 1\right] + 1}, \tag{77}$$

a result that follows from Proposition 1, as the ratio $\underline{z}/\underline{w}$ is constant with respect to the minimum wage.

The distribution of earnings in the formal sector is:

$$G^F(w) = \begin{cases} 0 & w < \underline{w} \\ \frac{L^{\underline{w}}}{1-L^I} & w = \underline{w} \\ \frac{L^{\underline{w}}}{1-L^I} + \frac{1-L^{\underline{w}}-L^I}{1-L^I}\cdot\left[1-(\underline{w}/w)^{\nu-\eta}\right] & w > \underline{w} \end{cases} \tag{78}$$

$$g^F(w)dw = \begin{cases} 0 & w < \underline{w} \\ \frac{L^{\underline{w}}}{1-L^I} & w = \underline{w}\,, \\ \frac{1-L^{\underline{w}}-L^I}{1-L^I}(\nu-\eta)\underline{w}^{\nu-\eta}w^{-(\nu-\eta+1)}dw & w > \underline{w} \end{cases} \tag{79}$$

so mean log earnings in the formal sector is:

$$\mathbb{E}^F[\log w] = \frac{\alpha}{\nu-\eta} + \log\underline{w}, \quad \alpha \equiv \frac{1-L^{\underline{w}}-L^I}{1-L^I} \tag{80}$$

and variance of log earnings in the formal sector is:

$$V^F(\log w) = \frac{2\alpha-\alpha^2}{(\nu-\eta)^2} \tag{81}$$

which implies that $\frac{\partial V^F}{\partial \underline{w}} = 0$.

Using the formulas for mean and variance of log earnings in the formal sector, rewrite the marginal effect of $\underline{w}$ on earnings inequality as:

$$\frac{\partial V}{\partial \underline{w}} = \frac{\partial L^I}{\partial \underline{w}} \left[\left(\mathbb{E}^I - \frac{\alpha}{\nu - \eta} - \log \underline{w} \right)^2 + V^I - \frac{2\alpha - \alpha^2}{(\nu - \eta)^2} \right]. \tag{82}$$

However, the distribution of earnings in the informal sector is truncated Pareto:

$$G^I(w) = \frac{1 - (w_0^I / w)^{\nu - \eta}}{1 - (w_0^I / w_1^I)^{\nu - \eta}}, \quad w \in [w_0^I, w_1^I] \tag{83}$$

so mean and variance of log earnings in the informal sector are:

$$\mathbb{E}^I(\log w) = \frac{(w_0^I)^{-(\nu-\eta)} \log(w_0^I) - (w_1^I)^{-(\nu-\eta)} \log(w_1^I)}{(w_0^I)^{-(\nu-\eta)} - (w_1^I)^{-(\nu-\eta)}} + \frac{1}{\nu - \eta} \tag{84}$$

$$V^I(\log w) = \frac{1}{(\nu - \eta)^2} - \left[\frac{\log(w_1^I / w_0^I)}{(w_0^I)^{-(\nu-\eta)} - (w_1^I)^{-(\nu-\eta)}} \right]^2 (w_0^I w_1^I)^{-(\nu-\eta)} \tag{85}$$

for $w_0^I = \frac{\eta}{\eta+1}(1-\rho)z_0$ and $w_1^I = \frac{\eta}{\eta+1}(1-\rho)\underline{z}$. When $\underline{w}$ is such that $w_1^I \approx w_0^I$ the mean and variance of log earnings in the informal sector are:

$$\mathbb{E}^I(\log w) = \log(w_0^I), \quad V^I = 0 \tag{86}$$

so the marginal effect of the minimum wage on variance of log earnings is:

$$\frac{\partial V}{\partial \underline{w}} = \frac{\partial L^I}{\partial \underline{w}} \left[\left(\log \left(w_0^I / \underline{w} \right) - \frac{\alpha}{\nu - \eta} \right)^2 - \frac{2\alpha - \alpha^2}{(\nu - \eta)^2} \right]. \tag{87}$$

To show that $\frac{\partial V}{\partial \underline{w}} > 0$ I then show that $\frac{\partial L^I}{\partial \underline{w}} > 0$ and $[..] > 0$.

The informal share of workers takes the form:

$$L^I = \frac{(w_0^I / \underline{w})^{-(\nu-\eta)} - (w_1^I / \underline{w})^{-(\nu-\eta)}}{(w_0^I / \underline{w})^{-(\nu-\eta)} - (w_1^I / \underline{w})^{-(\nu-\eta)} + \frac{\nu-\eta}{\nu} \left[(w_1^I / \underline{w})^{-\nu} - (1-\rho)^{-\nu} \right] + (1-\rho)^{-\nu}} \tag{88}$$

and define $x \equiv (w_0^I / \underline{w})^{-(\nu-\eta)}$, $A \equiv (w_1^I / \underline{w})^{-(\nu-\eta)}$, $B \equiv \frac{\nu-\eta}{\nu} \left[(w_1^I / \underline{w})^{-\nu} - (1-\rho)^{-\nu} \right]$,

and $C \equiv (1-\rho)^{-\nu}$, with A, B, and C being constants due to Proposition 1, to write:

$$L^I = \frac{x - A}{x - A + B + C} \tag{89}$$

and notice that because x increases with $\underline{w}$, I have $\frac{\partial L^I}{\partial \underline{w}} > 0$.

Lastly, I show that:

$$\left(\log\left(w_0^I/\underline{w}\right) - \frac{\alpha}{\nu - \eta}\right)^2 - \frac{2\alpha - \alpha^2}{(\nu - \eta)^2} > 0 \tag{90}$$

Open the quadratic term, invert w_0^I and $\underline{w}_0$, and put $\alpha/(\nu - \eta)$ in evidence:

$$\left(\log\left(w_0^I/\underline{w}\right) - \frac{\alpha}{\nu - \eta}\right)^2 - \frac{2\alpha - \alpha^2}{(\nu - \eta)^2} = \left(\log(\underline{w}_0/w_0^I)\right)^2 + \frac{2\alpha}{\nu - \eta}\underbrace{\left(\log(\underline{w}_0/w_0^I) - \frac{1-\alpha}{\nu - \eta}\right)}_{=T} \tag{91}$$

Notice that if T is positive, [..] is positive, and the variance of log earnings increases with the minimum wage at $\underline{w}_0$. Use the formula for α and define $x_0 = \underline{w}_0/w_0^F > 1$ to rewrite T as:

$$T = \log(x_0/(1-\rho)) - \frac{x_0^\nu - 1}{(\nu - \eta)x_0^\nu + \eta} \tag{92}$$

where x_0 is determined by equation:

$$\frac{(1-\rho)^{\eta+1}}{\eta + 1} + \frac{\eta}{\eta + 1}x_0^{\eta+1} - x_0^\eta = 0 \tag{93}$$

Importantly, x_0 is pinned down independently of ν (inspection). To show that $T > 0$, I will show that it increases with $\nu > \eta$, and that at $\nu = \eta$, $T > 0$. Differentiate it with respect to ν:

$$\frac{\partial T}{\partial \nu} = \frac{x_0^\nu\left(x_0^\nu - 1 - \nu \log x_0\right)}{\left[(\nu - \eta)x_0^\nu + \eta\right]^2} > 0 \quad \Leftrightarrow \quad \frac{x_0^\nu - 1}{\nu} > \log x_0 \tag{94}$$

However, because $\nu > \eta$, the above condition is satisfied (as $\log x \leq x - 1$):

$$\frac{x_0^\nu - 1}{\nu} > \frac{x_0^\eta - 1}{\eta} \geq \log x_0 \tag{95}$$

At $\nu = \eta$ the term becomes:

$$T(\nu = \eta) = \log(x_0/(1-\rho)) - \frac{x_0^\eta - 1}{\eta} \tag{96}$$

I want to argue that for any $\rho > 0$, $T(\nu = \eta) > 0$. To do so, I show that at $\rho = 0$, $T(\nu = \eta) = 0$ and that $T(\nu = \eta)$ increases strictly with ρ. It is easy to see that when $\rho = 0$ implies $x_0 = 1$ and $T(\nu = \eta) = 0$. The derivative is:

$$\frac{\partial T(\nu = \eta)}{\partial \rho} = \frac{1}{x_0}\left[\frac{\partial x_0}{\partial \rho}(1 - x_0^\eta) + \frac{x_0}{1-\rho}\right] \tag{97}$$

Differentiate the equation that defines x_0:

$$\frac{\partial x_0}{\partial \rho} = \frac{(1-\rho)^\eta}{\eta}\frac{1}{x_0^\eta}\frac{x_0}{x_0 - 1} \tag{98}$$

So:

$$\frac{\partial T(\nu = \eta)}{\partial \rho} = \frac{1}{1-\rho} - \frac{(1-\rho)^\eta}{\eta}\frac{1}{x_0 - 1}\frac{x_0^\eta - 1}{x_0^\eta} \tag{99}$$

Hence,

$$\frac{\partial T(\nu = \eta)}{\partial \rho} > 0 \quad \Leftrightarrow \quad \frac{(1-\rho)^{\eta+1}}{x_0^\eta}\frac{1}{\eta(x_0 - 1)}(x_0^\eta - 1) < 1 \tag{100}$$

But, from the definition of x_0:

$$\frac{(1-\rho)^{\eta+1}}{\eta + 1} + \frac{\eta}{\eta + 1}x_0^{\eta+1} - x_0^\eta = 0 \quad \Rightarrow \quad \frac{(1-\rho)^{\eta+1}}{x_0^\eta} = 1 - \eta(x_0 - 1) \tag{101}$$

Substitute to find:

$$\frac{1 - \eta(x_0 - 1)}{\eta(x_0 - 1)}(x_0^\eta - 1) < 1 \quad \Leftrightarrow \quad x_0^\eta < \frac{1}{1 - \eta(x_0 - 1)} \tag{102}$$

which is true since (again from definition of x_0, and $\rho > 0$):

$$x_0^\eta = \frac{(1-\rho)^{\eta+1}}{1 - \eta(x_0 - 1)} < \frac{1}{1 - \eta(x_0 - 1)} \tag{103}$$

E.4 Proof of Proposition 5

Fix ℓ and solve for $\{l_h(z), w_h(z)\}$ by minimizing production costs:

$$C^{inf}(\ell; z) = \min_{\{l_h(z), w_h(z)\}} \left\{ \sum_h w_h(z) l_h(z) \right\} \tag{104}$$

$$\text{s.t.} \quad \ell = \left[\sum_h \xi_h(z) l_h(z)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad l_h(z) = N_h \left[\frac{w_h(z)}{W_h} \right]^{\eta} \quad \forall h = 1, ..., H \tag{105}$$

Solving the above problem I calculate the labor allocated at skill group h by firm z as a function of aggregates (due to the monopsonistic competition assumption) and ℓ:

$$l_h(z) = \frac{\xi_h(z)^{\frac{\eta\varepsilon}{\eta+\varepsilon}} \left(N_h^{1/\eta} / W_h \right)^{\frac{\eta\varepsilon}{\eta+\varepsilon}}}{\left[\sum_{h'} \xi_{h'}(z)^{\frac{\eta\varepsilon}{\eta+\varepsilon} \frac{1+\eta}{\eta}} \left(N_{h'}^{1/\eta} / W_{h'} \right)^{\frac{\eta\varepsilon}{\eta+\varepsilon} \frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}} \ell \tag{106}$$

I now turn to determining the scale of operation of the firms. Substitute for the cost function, which becomes linear in the cost index for firm z: $\mathbb{W}(z)\ell^{\frac{\eta+1}{\eta}}$. The profit maximization problem then reads:

$$\pi^{inf}(z) = \max_{\ell} \left\{ (1-\rho) z \ell - \mathbb{W}(z) \ell^{\frac{\eta+1}{\eta}} \right\} \tag{107}$$

and the resulting aggregate labor and profits are, respectively:

$$\ell(z) = \left[\frac{\eta}{\eta+1} (1-\rho) \frac{z}{\mathbb{W}(z)} \right]^{\eta} \tag{108}$$

$$\pi^{inf}(z) = \mathbb{W}(z)^{-\eta} \frac{\eta^{\eta}}{(\eta+1)^{\eta+1}} (1-\rho)^{\eta+1} z^{\eta+1} \tag{109}$$

It is easy to see that the formulas in the proposition are just algebraic manipulations of the results derived in this proof.

E.5 Proof of Proposition 4

Substitute production function and CES labor aggregate constraints to find:

$$\pi^{form}(z) = \max_{\{w_h(z)\}_h} \left\{ z \left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^{\eta}} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^{\eta \frac{\varepsilon-1}{\varepsilon}} w_h(z)^{\eta \frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} - (1+\tau) \sum_h \frac{N_h}{W_h^{\eta}} (1+\varsigma_h)^{\eta} w_h(z)^{\eta+1} \right\} \tag{110}$$

$$w_h(z) \geq \underline{w} \quad \forall h = 1, ..., H \tag{111}$$

Let μ_h be the multiplier associated with the minimum wage constraint on skill h. The first order conditions are:

$$z \left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^{\eta\frac{\varepsilon-1}{\varepsilon}} w_h(z)^{\eta\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^{\eta\frac{\varepsilon-1}{\varepsilon}} \eta w_h(z)^{\eta\frac{\varepsilon-1}{\varepsilon}-1} + \mu_h = (1+\tau)\frac{N_h}{W_h^\eta}(1+\varsigma_h)^\eta(\eta+1)w_h(z)^\eta \tag{112}$$

$$\mu_h(w_h(z) - \underline{w}) = 0, \quad \mu_h \geq 0 \quad \forall h = 1, ..., H \tag{113}$$

If the wage offer is greater than the minimum wage then $\mu_h = 0$ and the first order condition becomes:

$$\frac{\eta}{\eta+1}\frac{z}{1+\tau}\left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^{\eta\frac{\varepsilon-1}{\varepsilon}} w_h(z)^{\eta\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{-\frac{1}{\varepsilon}} (1+\varsigma_h)^{-\frac{\eta}{\varepsilon}} w_h(z)^{-\frac{\eta}{\varepsilon}-1} = 1 \tag{114}$$

Define $S \equiv \left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^{\eta\frac{\varepsilon-1}{\varepsilon}} w_h(z)^{\eta\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \geq 0$ and notice that, for all $h = 1, ..., H$, optimal wages are a function of S:

$$w_h(z) = \max\{f_h(S;z), \underline{w}\}, \quad f_h(S;z) \equiv \left[\frac{\eta}{\eta+1}\frac{z}{1+\tau}\xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{-\frac{1}{\varepsilon}} (1+\varsigma_h)^{-\frac{\eta}{\varepsilon}} \right]^{\frac{\varepsilon}{\eta+\varepsilon}} S^{\frac{\varepsilon}{\eta+\varepsilon}} \tag{115}$$

Lastly, define the operator $T : \mathbb{R}_+ \mapsto \mathbb{R}_+$ as:

$$T(S) = \left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^{\eta\frac{\varepsilon-1}{\varepsilon}} \max\{f_h(S;z), \underline{w}\}^{\eta\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \tag{116}$$

where finding equilibrium wages boils down to finding a fixed poing S^* such that $T(S^*) = S^*$, and then recovering wages through equation (115). Importantly, it is easy to see that $T(S)/S$ is continuous for all $S \geq 0$, it decreases with S, that $\lim_{S\to 0} T(S)/S = \infty$ and $\lim_{S\to\infty} T(S)/S = 0$. The above conditions imply that there exists a unique S^* such that $T(S^*) = S^*$, i.e., a unique solution to the problem of the formal firm.

F Walras' Law

This Appendix calculates the goods' market clearing. Aggregate demand for goods read:

$$C = \sum_{h=1}^{H} \int_{i \in h} \int_{j \in \Omega} Prob_{ih}(j) c_{ih}(j) dj di \tag{117}$$

From the household problem $c_{ih}(j) = w_h(j)$ and conditional on h and the firm j the household is working for, all individual i's are symmetric (that is: $Prob_{ih}(j) = Prob_h(j)$). This implies:

$$C = \sum_{h=1}^{H} \int_{j \in \Omega} N_h Prob_h(j) w_h(j) dj \tag{118}$$

From the structure of the problem, in equilibrium I have that the labor demand for skill h by firm j equals the fraction of households of skill h that are choosing to work in that firm: $l_h(j) = N_h Prob_h(j)$, hence:

$$C = \int_{j \in \Omega_{form}} \sum_{h=1}^{H} l_h(j) w_h(j) dj + \int_{j \in \Omega_{form}} \sum_{h=1}^{H} l_h(j) w_h(j) dj \tag{119}$$

where I also inverted the order of the summation and split firms into informal and formal sectors. Each term in turn becomes:

$$\int_{j \in \Omega_{form}} \sum_{h=1}^{H} l_h(j) w_h(j) dj = \int_{j \in \Omega_{form}} q(j) dj - \int_{j \in \Omega_{form}} \pi^{form}(j) dj \tag{120}$$

$$\int_{j \in \Omega_{inf}} \sum_{h=1}^{H} l_h(j) w_h(j) dj = (1-\rho) \int_{j \in \Omega_{inf}} q(j) dj - \int_{j \in \Omega_{inf}} \pi^{inf}(j) dj \tag{121}$$

This implies that the goods market clearing condition:

$$C + \Pi^{form} + \Pi^{inf} + \rho Q^{inf} = \int_{j \in \Omega} q(j) dj = Q \tag{122}$$

which states that total production is split into consumption, profits for formal and informal firms, and government collection of revenue due to fiscalized informal units.

G Computation of the quantitative model

This Appendix details the computation of the quantitative model.

I discretize the state space of firms. For the first productivity component, which follows a log-Normal, I construct a grid between 0.0001 and 100 with 112 points concentrated over 1 (call it $\nu \in \mathcal{V}$). For the second productivity component, which follows a Pareto, I assume a grid between 1 and 100 with 93 points also concentrated over 1 (call it $\theta \in \Theta$). Hence, the final grid for firm-level productivities has 10,416 points, consisting of all possible combinations of points in the two grids above: $z \in \mathcal{Z} = \mathcal{V} \times \Theta$.

To solve the model computationally, I use the following algorithm:

1. Guess a vector of wage indices W_h^0 for $h = 1, ..., H$

2. Calculate formal and informal profits, labor, and wages over for each firm productivity $z \in \mathcal{Z}$ following the details in the main text.

3. Update the vector of wage indices:

$$W_h^1 \equiv \left[\sum_{z \in \mathcal{Z}} (1 + \varsigma_h(z))^\eta w_h(z)^\eta \right]^{\frac{1}{\eta}}, \quad \forall h = 1, ..., H$$

4. Compare W_h^0 and W_h^1 for all h, update and iterate until convergence

H Kaitz analysis

This section leverages on variation at the state level over time to correlate the increases in the minimum wage with the earnings inequality in the formal sector, informal sector, and in the aggregate. There are three main takeaways: first, the results suggest that inequality in the formal sector falls with the minimum wage. Second, an increase in the minimum wage correlates positively with inequality in the informal sector and the informal share of labor. Third, and as a consequence, the reduced-form relationship between the minimum wage and aggregate inequality is negative, but smaller in magnitude than the relationship with formal sector inequality.

Differently to the estimation strategy in the main text, the analysis in this appendix closely follows the empirical framework and methods in Autor et al. (2016).[22] I use the log-distance between the minimum wage and the median wage in the formal sector (also known as the Kaitz index) as a measure for how restrictive the minimum wage is for state s in year t:

$$\text{Kaitz}_{st} \equiv \log\left(\frac{w_t}{w_{st}^{50,F}}\right). \tag{123}$$

I correlate the minimum wage with different measures of earnings inequality (y_{st}) by regressing:

$$y_{st} = \beta_1 \cdot \text{Kaitz}_{st} + \beta_2 \cdot \text{Kaitz}_{st}^2 + \alpha(s,t) + \varepsilon_{st}, \tag{124}$$

where $\alpha(s,t)$ represents controls at the state and year level. These controls absorb state and national-level changes in the shape of the wage distribution that are not related to the minimum wage. I also experiment controlling for the unemployment rate in state s time t as a proxy for heterogeneous shocks to a state's labor market.[23] My preferred specification follows Engbom and Moser (2021) and includes state fixed effects and state-specific quadratic time trends, even though I display the results for a variety of different controls. The identification assumption is that, conditional on $\alpha(s,t)$, the error term ε_{st} is uncorrelated with the Kaitz index. Identification of β_1 and β_2 comes from movements in the minimum wage that deviate from state-

[22] See Lee (1999), Haanwinckel (2020), and Engbom and Moser (2021) for papers with similar specifications.

[23] For instance, Costa et al. (2016) and Adão (2016) study the regional effects of the commodity boom in 2000s Brazil.

Table H.1: Reduced-form evidence on the effects of the minimum wage

Outcomes	Formal	Informal	Aggregate
log(Variance)	-0.985***	0.172**	-0.151*
	(0.085)	(0.081)	(0.076)
log(Informal share)		0.162***	
		(0.051)	

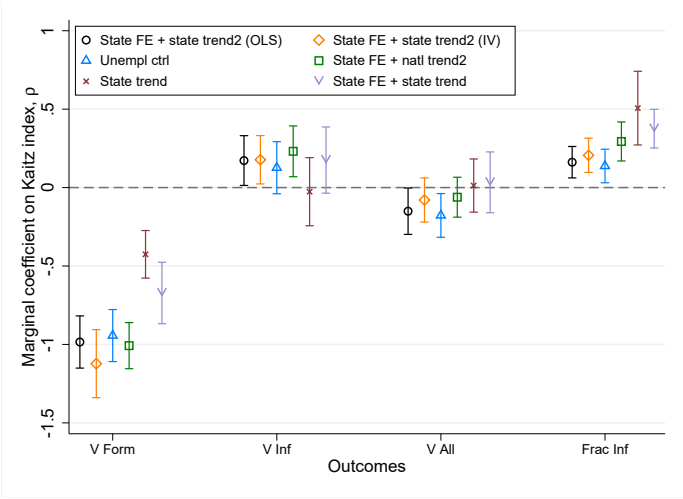
Notes: Each cell represents a separate regression. Each cell reports the marginal coefficient on the minimum wage ($\rho = \hat{\beta}_1 + 2\hat{\beta}_2\overline{kaitz}$), where the β-coefficients are obtained by regressing (124). All specifications control for state fixed effects and state-specific quadratic time trends. Marginal coefficients are evaluated at median wage. All regressions are employment-weighted and have 405 observations (27 states by 15 years). Standard errors in parentheses are clustered at the state level. *Sources:* 1996/2012 PNAD.

specific quadratic time trends. The marginal coefficient on the minimum wage, the object displayed in the figures that follow, is estimated as: $\rho = \hat{\beta}_1 + 2\hat{\beta}_2\overline{kaitz}$, and I evaluate it at the employment-weighted median Kaitz index.

Table H.1 reports the estimated relationships between the minimum wage and different outcomes, ρ. Each row corresponds to a different inequality measure, and each column corresponds to a specific distribution of earnings. The first column displays the results for the formal earnings distribution, the second column displays the results for the informal earnings distribution, and the last column discusses the results for the aggregate earnings distribution. The last row calculates the relationship between the minimum wage and the informal share of labor. There is a negative and significant relationship between the minimum wage and formal inequality (-0.985***). Importantly, these regression estimates are consistent with other evidence for Brazil (Engbom and Moser (2021) and Haanwinckel (2020)).

The second column estimates the same set of regressions, but focuses on the informal earnings distribution. There is a significant relationship of 0.172** between the minimum wage and variance of informal earnings. Moreover, the last row in Table H.1 reports that increases in the minimum wage are associated with increases in the informal share of labor (0.162***). Hence, either through the movement of more productive workers from formal to informal jobs (Jales, 2018), or through competition effects in the labor markets (Derenoncourt et al., 2021), the reduced-form evidence suggests that there exists a non-trivial relationship between the minimum wage and the informal sector.

Figure H.1: Marginal effect of the minimum wage (alternative specifications)



Notes: Plot shows the marginal effect of the minimum wage on different outcomes (x-axis) for different specifications (colors). "State trend" denote state-specific linear time trends, "state trend2" denote state-specific quadratic time trends, and "natl trend2" denote a national quadratic time trend. *Sources:* 1996-2012 PNAD.

Lastly, the third column looks at the association between the minimum wage and aggregate distribution of earnings. This takes into consideration not only the within-sector associations discussed above, but also how the minimum wage is related to the distance between mean earnings in the formal and informal sectors. The relationship between the minimum wage and aggregate inequality is negative, but less significant and smaller in magnitude than the relationship with formal sector inequality (-0.151* vs. -0.985***, respectively), due to the counteracting forces presented by the informal sector earnings distribution.

I now discuss the robustness of these results. Figure H.1 shows that the results are robust to different specifications of the control variables: controlling for unemployment rate, state-specific linear time trends, no state fixed effects as in Lee (1999), state-specific linear time trends and national quadratic time trends as in Haanwinckel (2020), among others. I use two different strategies to control for the possibly mechanical endogeneity of the Kaitz index, as it might correlate with the residual term because median wages might affect the dispersion in earnings. First, I redo the analysis with the share of formal workers at the minimum wage as the measure for how binding the minimum is in a given state-year. Second, I follow the 2SLS IV approach from Autor et al. (2016), where the first stage projects the

Table H.2: Reduced-form evidence on the effects of the minimum wage (share of minw formal workers as main measure)

Outcomes	Formal	Informal	Aggregate
log(Variance)	-1.382***	0.897***	0.730***
	(0.264)	(0.286)	(0.210)
log(Informal share)		0.445**	
		(0.208)	

Notes: Each cell represents a separate regression. Each cell reports the marginal coefficient on the minimum wage on the regression: $y_{st} = \beta \cdot atminw_{st} + \alpha_s + \alpha_t + \varepsilon_{st}$. All regressions are employment-weighted and have 405 observations (27 states by 15 years). Standard errors are clustered at the state level. *Sources:* 1996/2012 PNAD.

Kaitz index and its square on log minimum wage, its square, and its interaction with the state's overall median earnings throughout the sample period, thus filtering for transitory shocks on median wages. The results are similar and displayed in Table H.2 and Figure H.1, respectively. Lastly, Figure H.2 compares the estimates of the effect of the minimum wage on informal share of labor with those found at Engbom and Moser (2021), and shows that if we apply similar sample restrictions I also obtain a null relationship between the minimum wage and informal share.

Figure H.2: Comparison with Engbom and Moser (2021)

Notes: This figure plots the estimated marginal effect of the minimum wage on informal share of labor for different specifications. The first row ("Engbom Moser 2021") shows the weakly positive effect of minimum wage on formal share, taken from Engbom and Moser (2021). The second row is a replication attempt of the RAIS data set with the PNAD data set. The third row includes female workers in the sample. The "Self empl" row excludes self employed workers from the sample. "Both" considers both male and female and excludes SE workers - which corresponds to the main specification in this paper. "Both+IV" uses the 2SLS strategy in Autor et al. (2016). *Sources:* 1996-2012 PNAD.

I Unemployment

In this Appendix, I add unemployment to the quantitative model, following Caliendo et al. (2019). After re-calibrating the model and performing the same set of counterfactuals, I show that still the most important margin of adjustment in response to the minimum wage is between the formal and informal sectors, not between employment and unemployment.

Households now face unemployment benefits b on top of firm-specific wage offers. Importantly, households draw a taste shock for being unemployed, which is also Fréchet distributed and independent from other firm-specific shocks. This formulation implies that a share $U_h = N_h \left(\frac{b}{W_h}\right)^{\eta}$ of households of skill h will opt out of the labor force. The aggregate wage indices in the economy now read:

$$W_h = \left[b^{\eta} + \int_{j \in \Omega} [(1 + \varsigma(j))w(j)]^{\eta} dj \right]^{\frac{1}{\eta}} \tag{125}$$

The problem of the firm does not change, so I do not discuss it in this Appendix. An equilibrium in this model consist of wage indices that equate aggregate labor supply and demand for each skill. The calibration procedure is identical to the one in the main specification, with the addition of the unemployment benefit parameter b, which is endogenously chosen to match aggregate unemployment $U = \sum_h U_h$ in 1996 and 2012. Table I.1 describes the parameters of this alternative calibration procedure, and Table I.2 compares the aggregate moments in the model with the data. The results of the calibration procedure are similar to those displayed in the model without unemployment.

Table I.3 studies the consequences of an increase in the minimum wage, keeping all other parameters constant at their 1996 levels. Notice that there are still unintended consequences of the minimum wage on overall earnings inequality, and that these appear because of strong informal sector responses. At the same time, increasing the minimum wage increases the unemployment rate by 9%. Intuitively, when firms become informal, they sharply reduce their labor demand. This "labor surplus" that is generated is only partially reallocated towards incumbent firms, and a share of these workers actually decide to drop out of the labor force, increasing the unemployment rate. Because the unemployment rate is low in the data (around 6%), the calibrated value of the unemployment benefit b is relatively small, and does not compete away workers from incumbent firms in the economy. Hence, even in a

Table I.1: Calibration results: model with unemployment

Parameter	Description	1996	2012	Target
$\underline{w}$	Minimum wage	3.985	8.623	Share at min wage
ρ	Probability of detection	0.269	0.327	Informal share
b	Unemployment benefits	0.620	1.237	Unemployment rate
ϕ_4		0.089	0.118	Relative wages (terc/sec)
ϕ_3	Demand shifter parameters	0.014	0.044	Relative wages (sec/prim)
ϕ_2		-0.027	-0.046	Relative wages (prim/no deg)
η	Labor supply elast.	4.856	4.121	Formal wage premium
σ	Standard deviation	0.957	1.324	Formal inequality
κ	Pareto tail	6.523	6.236	Informal inequality

Table I.2: Data vs. model comparison: model with unemployment

	1996		2012	
	Data	Model	Data	Model
Mean earnings				
Formal/Informal	2.06	2.13	1.65	1.68
Primary/No degree	1.39	1.39	1.19	1.20
Secondary/Primary	1.46	1.45	1.21	1.21
Tertiary/Secondary	2.49	2.49	2.15	2.15
Variance of log-earnings				
Overall	0.78	0.73	0.50	0.46
Formal	0.65	0.52	0.33	0.33
Informal	0.66	0.66	0.62	0.51
Formal bunching at min wage	0.077	0.077	0.158	0.159
Informal share of labor	0.390	0.386	0.299	0.299
Unemployment share	0.065	0.065	0.062	0.062

model with unemployment, the predominant margin of adjustment of the economy to an increase in the minimum wage is still between the formal and informal sectors.

Table I.3: The unintended consequences of the minimum wage in a model with unemployment

		All parameters at 1996 values, except...		
		$\Delta\underline{w} = 120\%$	$\Delta\rho = 23\%$	N_h
	1996	(minimum wage)	(enforcement)	(skill comp)
V(log earnings)				
overall	0.73	0.81	0.72	0.71
formal	0.52	0.43	0.52	0.56
informal	0.66	0.81	0.65	0.58
Fraction at $\underline{w}$	0.077	0.211	0.072	0.045
Informal share	0.390	0.871	0.275	0.205
Unemployment rate	0.065	0.071	0.073	0.016

J Discussion of Derenoncourt et al. (2021)'s findings

In this Appendix, I conciliate my findings to those in Derenoncourt et al. (2021). I confirm that specifications that compare states above and below median treatment do not estimate informality responses to the minimum wage. I estimate the following regression:

$$y_{st} = \alpha + \beta \cdot \text{Treated}_s \cdot \mathcal{I}_{t>1999} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}, \tag{126}$$

where y denotes the outcome of interest for state s state and year t and Treated_s is an indicator for states that are highly treated (i.e., above median treatment) by the minimum wage in the pre-period average. I include the same set of fixed effects and controls as in Table A.3. Moreover, I closely follow Derenoncourt et al. (2021) in including self-employed workers in the definition of informality.[24]

Table J.1 shows the difference-in-differences coefficients for two treatment measures. Columns (1) and (3) confirm the findings in Derenoncourt et al. (2021) that, relative to below-median treatment states, above-median treatment states did not experience a larger increase in the share of the informal sector after 1999. Column (1) uses the treatment measure in this paper and Column (2) uses the Kaitz index as an alternative measure. Specifications (2) and (4) highlight that treatment heterogeneity is important when analyzing the informality effects of the minimum wage.

[24] Results in this Appenix are invariant to whether or not I include self-employed individuals in the analysis.

Table J.1: Exploring treatment heterogeneity

	Bunching at $\underline{w}$		Kaitz index	
	(1)	(2)	(3)	(4)
β	0.0217		0.0161	
	(0.0176)		(0.0180)	
β_2		0.0513*		0.0679**
		(0.0261)		(0.0252)
β_3		0.0184		0.0498*
		(0.0437)		(0.0247)
β_4		0.0457		-0.00613
		(0.0341)		(0.0337)
β_5		-0.0164		-0.0210
		(0.0218)		(0.0193)
β_6		0.0206		0.0265
		(0.0237)		(0.0221)
β_7		0.0259		0.0697***
		(0.0293)		(0.0201)
β_8		0.0695***		0.0625**
		(0.0203)		(0.0240)
β_9		0.0628**		0.0212
		(0.0244)		(0.0275)
Observations	405	405	405	405
R-squared	0.973	0.976	0.973	0.977

Notes: The first two columns measure treatment to the minimum wage using the fraction of workers bunched at the minimum before 1999. The latter two columns use the same definition of treatment as Derenoncourt et al. (2021): the Kaitz index. The dependent variable is the log of the informal share in each state. All regressions include self-employed workers in the definition of informality. Standard errors in parentheses clustered at state level. ***p<1%, **p<5%, *p<10%. *Sources:* 1996/2012 PNAD.