Social Security Reform, Retirement and Occupational Behavior*

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Abstract

In most countries, the rules governing public and private pension systems are different, and so are hiring procedures, and job contracts. The tenures of government employees are longer and their wages, in general, higher. In this sense, social security reforms will affect not only the decision to leave the labor force, but also the choice of which sector to work. In this article, we study the impact of social security reforms on retirement and occupational behavior. We develop a life-cycle model with three sectors - private formal, private informal and public - and endogenous retirement to evaluate what are the macroeconomic and occupational impacts of social security reforms in an economy with multiple pension systems. In a model calibrated to Brazil, we simulate and quantitatively assess the long-run impact of reforms being discussed and/or implemented in different economies. Among them, the unification of pension systems and the increase of the minimum retirement age. These reforms are found to affect the decision to apply to a public job, savings during the life cycle and skill composition across sectors. On the long run, they lead to higher output and capital, less informality and to average welfare gains. They also drastically reduce social security deficit.

Key Words: social security reform, public employment, public deficit, informality

JEL Codes: J26, H55, J45, J62

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1 Introduction

This paper explores quantitatively the effects of social security reforms on occupational and retirement behavior. This is a timely issue as several countries should face – or are already facing - problems in their social security systems due to ageing population and the financial fragility of their current systems. In particular, we study the effects of changes in social security regimes with different rules for public and private retirees on individual and aggregate behavior.

Population ageing is the result of increasing life expectancy and falling fertility rates. According to the United Nations (2013), life expectancy around the world has been increasing steadily in the last 15 years. In 2000, life expectancy at birth was 65 years old. Nowadays, an individual is expected to live 70.5 years. Old age support ratio[1] is expected to decrease from 3.8 nowadays to 2.1 in 2050 and 1.6 in 2100 for the OECD countries. On top of that, Blöndal and Scarpetta (1999) and Gruber and Wise (2002) provide evidence that the workforce participation of the elderly population has declined in many OECD countries.

Pay-as-you-go (PAYG) social security systems exist in most of the countries in the world. Some of them have differentiated pension rules for public servants. Kings et al. (2007) document that over half of the OECD members have different pension rules for public and private workers. For instance, the US has segregated pension plans, depending on the jurisdiction. Beshears et al. (2011) report that the majority of public pension schemes are still defined-benefit, unlike the private sector retirement plans.

Holzmann and Hinz (2005) argue that most public pension systems are not financially sustainable. Of course, taking into account sustainability issues, government's budgetary deficit problems are even more problematic. An important example is the Brazilian crisis in the aftermath of the East Asian and Russian financial meltdowns, in 1998. It was documented that a fiscal deficit of more than 6% of the GDP triggered this crisis, and that two thirds of this deficit was due to the cost of pensions. Likewise, in Lebanon, public retirees' pensions is the third greatest expenditure item in the government's budget, even though they account for less than 3% of the population.

Given the upcoming demographic pressures and the financial situation of social security systems, it is important to evaluate the long run consequences of potential reforms[2]. To provide a tool for the evaluation of the consequences of these reforms, we build a life-cycle model with heterogeneous agents and endogenous occupational and retirement decisions. In particular, we construct an economy with three working sectors: private formal, private informal and the government, each with its own pension system. We calibrate our economy to match key aspects of the Brazilian economy in 2013. With rapid ageing population and over generosity of its pension system – specially that of

[1] The ratio of individuals aged between 20 and 64 years old and those with more than 65 years old.

[2] See OECD (2015) for a summary of what reforms are being implemented around the world.

public workers – this country can be seen as a typical, if not exaggerated, case of the social security problems countries are currently facing, or will face in the near future. The calibrated model reproduces closely data regarding sectoral and labor decisions, retirement claims characteristics, as well as social security deficit and public job application decision.

We run a group of exercises to evaluate the log-run effects of social security reforms that are being discussed or implemented in economies around the world. First, we evaluate the effects of a reform introduced in 2013, that ended the "integrality" provision for public retirees, imposing a cap in their benefit and approximating the public and private social security system. This is similar, for instance, to the recent project proposed by the Macron administration in France of a universal retirement system that eliminates special regimes for the public sector. Second, we evaluate current proposals of increasing minimum retirement age.

Following the approach developed by Huggett (1996) to model life-cycle economies with heterogeneous agents in general equilibrium, Huggett and Ventura (1999), Conesa and Krueger (1999), among many, studied potential social security reforms and their macroeconomic consequences. These articles treat retirement exogenously. This is not a plausible assumption to address early retirement provisions[3], for instance. Imrohoroglu and Kitao (2012), Ferreira and dos Santos (2013), Jung and Tran (2012) and Gustman and Steinmeier (2005), among others, deal with endogenous retirement. None of these studies addresses retirement choices when agents face more than one working sector, however. This is important as social security reforms can induce workers to reallocate across different sectors, therefore having significant macroeconomic consequences such as fiscal deficit reduction.

If the unification of pension systems implies fewer benefits to public workers, high skill individuals may now opt for the private sector, increasing overall productivity of the economy. Segregated social security systems and the case of Brazil are dealt in Glomm et al. (2009), dos Santos and Pereira (2010), Dos Reis and Zilberman (2014) and dos Santos and Cavalcanti (2015). These articles emphasize how an overpaid and secure (in terms of job stability) public sector attracts the best human capital in the economy, and the macroeconomic consequences of such sector. What this literature have not done yet, as we do, is to develop a model to study occupational and retirement choices of agents that face multiple working and retirement sectors.

The remaining of the paper proceeds as follows. Section 2 establishes key facts about social security systems around the world, as well as some peculiarities of the Brazilian system. Section 3 presents the model, including the problem of the agents, the stationary distribution, and the equilibrium concept we use. Section 4 describes data used and the calibration procedure. Section 5 validates our numerical solution, comparing non-targeted moments generated by the model to the data. In Section 6, we evaluate the steady state macroeconomic consequences of different social security

[3]See Vestad (2013) for a list of countries that have early retirement provisions.

reforms and assesses the long-run welfare impacts of these reforms. Finally, Section 7 concludes.

2 Empirical Motivation

This section uses data from different sources to motivate why quantifying the macroeconomic consequences of social security reforms is important for governments around the world. We proceed in three steps. First, we show that population ageing and the current situation of pension expenditures will tighten the governments' budget in the near future. Second, we note that countries tend to have segregated retirement systems for private and public workers, and that understanding the consequences of reforms that unify them is important. Third, we detail the Brazilian situation, explain why we use this country as a case study, and highlight some aspects of the Brazilian economy that guide our theoretical and quantitative analysis.

Population ageing is a widespread phenomenon that governments throughout the world will have to deal within the next decades. According to OECD data, the share of the elderly in the population is expected to increase significantly until 2100. For the OECD countries, the share of individuals over seventy years of age is expected to increase by almost 70% until 2050, from 12.2% to 20.6%. Considering the world as a whole, this share is expected to move from 5% in 2015 to 11% in 2050.

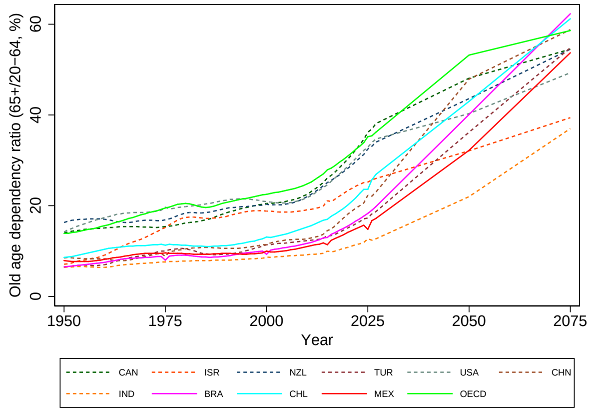
The increase of the elderly population will be followed by a likely reduction in the proportion of working age individuals in the future. Figure 1 shows that countries such as Brazil, Mexico and Chile expect large increases in the old age dependency ratio, the ratio between the retired population (over 65 years old) to the working-age population (20-64 years old). Old age dependency ratio is an important indicator when considering the financial sustainability of social security systems, as most countries adopt PAYG schemes, in which intergenerational transfers between the working population and the retired population occur.

Government expenditures on retirement benefits constitute a large share of GDP. For a selected group of countries, the average share is projected to increase 17% by 2050, going from 8.1% to 9.5%, as Figure 2 shows. The average increase across the sample is 44%. The demographic change and higher expenditures with the elderly population suggest that the social security deficit will worsen in the future. Therefore, social security reforms ought to be discussed and implemented all over the world, as documented by OECD (2015).

Several countries have retirement systems that vary according to the employment sector. In particular, public employees tend to have better retirement conditions relative to their private sector peers[4]. On average, public sector employees' pensions account for 20% of total pensions expen-

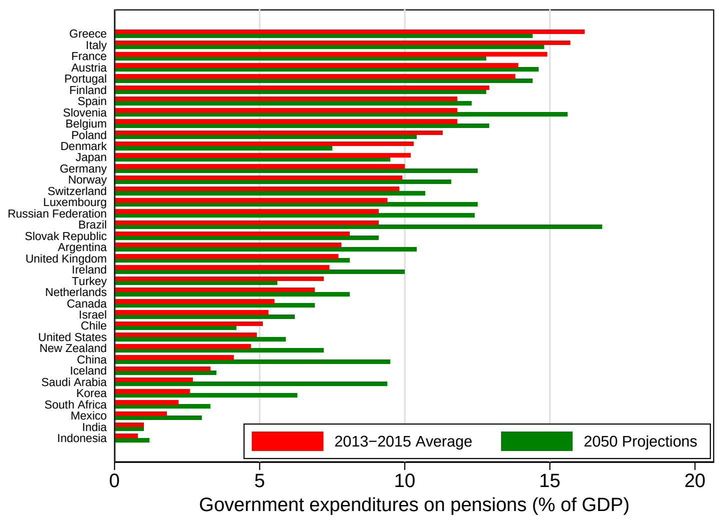
[4]See OECD Pensions Outlook (2018) for a detailed analysis on the countries in which the retirement conditions for public and private workers are different.

Figure 1: The evolution of the old age dependency ratio



Notes: This figure plots the evolution of the fraction of individuals aged 65+ to those aged 20-64 years old across different countries in the world. Source: 2017 Pensions at a Glance (OECD).

Figure 2: Public expenditure on retirement pensions

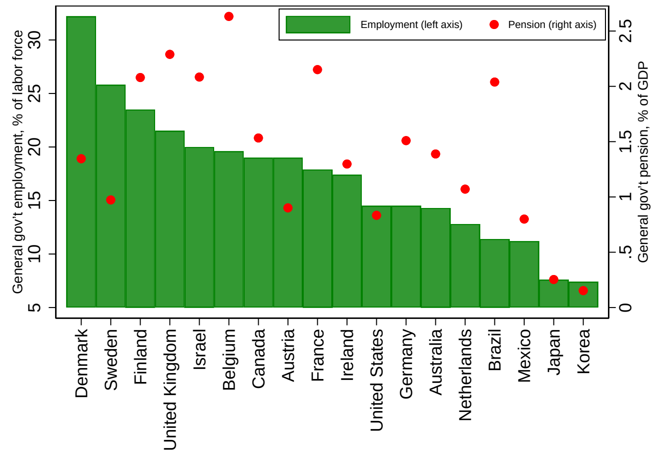


Notes: This figure plots the share of GDP governments spent on retirement pensions across 2013-2015, and those projected for 2050. Sources: 2017 Pensions at a Glance (OECD).

ditures[5]. Furthermore, these pensions represent a large share of output. Figure 3 shows that the general government takes up, on average, 17% of the labor force, and that their retirement benefits sum up to 1.5% of GDP. Hence, it is not surprising that there are a large number reform proposals that focus on the unification of the retirement schemes for private and public workers - e.g., in France and Brazil - and they will be one of the focus of our analysis.

[5]This number was calculated by matching the data from Figures 2 and 3.

Figure 3: General government size and pensions



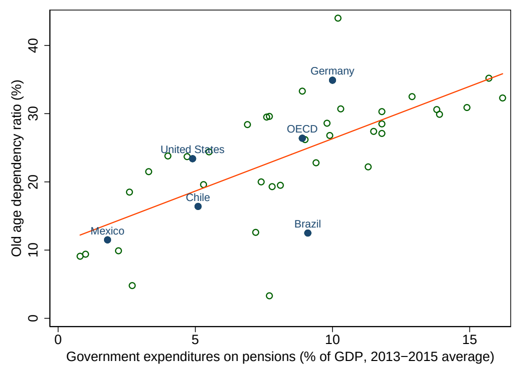
Notes: This figure plots the share of GDP governments spend on general government retirees (right) and the size of the general government in the labor force (left) in 2013. Sources: Social Security Expenditure Database (OECD), Brazilian National Accounts and Annual Report on Social Security (IBGE and Ministry of Social Security), and ILOSTAT Database (ILO).

The quantitative part of the paper specializes to the case of Brazil. We believe Brazil is representative of countries that already face or will face problems in their social security system for three reasons. First, Brazilian demographics follow global trends. This country's life expectancy went from 54.7 years in 1960 to 73.6 in 2012, and the Brazilian Institute of Geography and Statistics' (henceforth IBGE) projects that the share of individuals aged 65 and over will jump from 4% in 1980 to 22% around 2050.

Second, Brazil devotes a large share of its output to overall retirement pensions given its current demographics. Figure 4 plots the cross-country relationship between the 2013-2015 average share of GDP that retirement pensions represent and the old age dependency ratio in 2013. One can see that the Brazilian economy is as "young" as Chile and Mexico but spends a similar share of GDP with social security as Germany and the OECD average, where the population is considerably older.

Third, Figure 3 shows that the Brazilian economy has the widest discrepancy between the government size, 11.4% of the labor force, and the size of government retirees' pensions, 2% of GDP. Moreover, Table 1 shows that the social security deficit associated with both private and public workers' pensions has been systematically increasing over the last years, with similar contributions from the private and public social security regimes. However, the number of retirees from the public sector, around one million people, is much smaller than those from the private sector, around 29 million people.

Figure 4: The cross-section of demographics and pensions expenditure



Notes: This figure shows the cross-country relationship between the old-age dependency ratio (ratio of 65+ to 20-64 years old individuals) and the government expenditures on pensions as a share of GDP. Sources: 2017 Pensions at a Glance (OECD).

Table 1: Brazilian social security deficit (% GDP), 2013-2016

	Private	Public	SS Deficit
2013	0.94	1.18	2.12
2014	1.00	1.18	2.19
2015	1.44	1.22	2.66
2016	2.17	1.18	3.35

Notes: This table shows the evolution of the social security deficit as a share of GDP by sector. Source: Tafner et al. (2015).

We now describe some facts about the Brazilian economy used to guide our quantitative exercise. First, the lack of a minimum retirement age in the private sector makes early retirement a prevalent feature of the Brazilian retirement system. Table 2 shows that the average age individuals claimed for retirement benefits under the contribution modality[6] is 56 years old for men and 52 for women. The expected duration of retirement is 23 years for men and 29 for women. The discrepancy between these numbers relative to the rest of the world is large. For example, the OECD's average retirement age is 64, with an expected duration of 16 years for men.

Second, government sector jobs in Brazil tend to be relatively more stable and well-paid. Using data from the 2013 *Pesquisa Nacional por Amostra de Domicílios* (PNAD, a household survey), Figure 5 plots the job tenure of private and public workers. The average tenure of a private sector job is 5.3 years, whereas the average tenure for a public sector job is 13.6 years. On top of that, using data from the 2012-2018 *Pesquisa Nacional por Amostra de Domicílios Contínua* (PNADC)[7], we find evidence of a public sector wage premium. We regress log hourly earnings on a rich set of

[6]This is a type of retirement Brazilians can opt to have, and it will be further detailed in the next sections.
[7]See Section 4 for more details on the PNAD and the PNADC data sets.

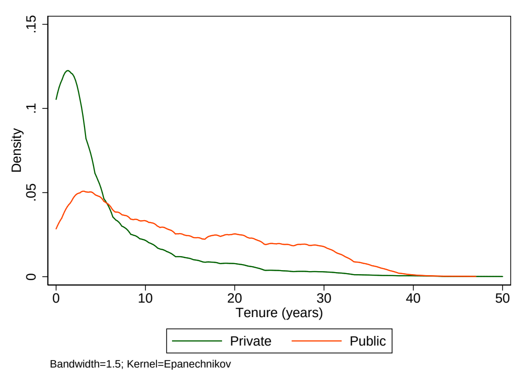
Table 2: Early retirement in Brazil, 2010

	Retirement Age		Expected Duration	
Location	Men	Women	Men	Women
OECD	64	63	16	21
Latin America	62	60	17	21
World	62	60	16	21
Brazil:				
Private: Contribution modality	56	52	23	29
Private: Age modality (Rural)	60	55	19	26
Private: Age modality (Urban)	65	60	16	22

Notes: This table compares the average retirement age and the expected duration of retirement across countries and different retirement modalities in Brazil in 2010. Source: Tafner et al. (2015).

controls and individual-level fixed effects, and estimate a public sector wage premium of 9.4% (see Table 4).

Figure 5: Estimated tenure density of public and private jobs, 2013



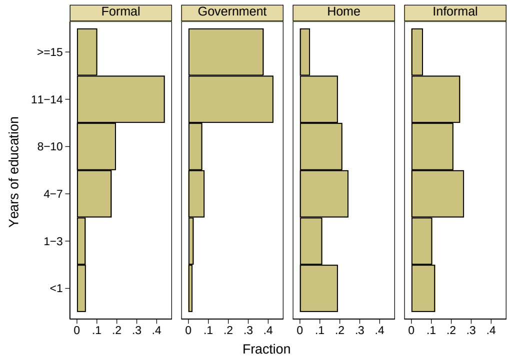
Notes: This figure calculates the density of the on-the-job tenure for public (red) and private (green) workers. We restrict our sample to male employees between 16-75 years old. Sample weights are used. Source: 2013 PNAD (IBGE).

The two facts above, together with a more generous retirement scheme and a competitive hiring process[8], make the public sector particularly attractive for more educated individuals. Even though the statutory public servants account for only 5% of the population, they represent nearly 20% of all individuals with more than 15 years of education. Moreover, the group of individuals that took the public exam (i.e., those who are trying to enter the public sector) represents 6% of the total population, but accounts for 23% of all individuals with more than 15 years of education[9].

[8]See Section 4 for details on the Brazilian retirement system, and Section 3 for a discussion on the public employment hiring procedure.

[9]These numbers were calculated using the 2013 PNAD data.

Figure 6: Distribution of years of education by sector, 2013



Notes: This figure plots the histogram of years of education across the formal, government, home and informal sectors. We define the home sector as those individuals that are not occupied by the time of the survey. We restrict our sample to male employees between 16-75 years old. Sample weights are used. Sources: 2013 PNAD (IBGE)

Third, the Brazilian economy has a large share of informal workers that do not pay most taxes and are not under the protection of the Social Security Administration[10]. According to the 2013 PNAD data, 18% of males with 16 to 75 years old are in the informal private sector[11]. Because of the lack of job security and the type of employers that are typically hiring in the informal sector[12], it ends up absorbing individuals that are on average less educated and consequently end up receiving lower incomes. This evidence is displayed in Figure 6, which shows the distribution of agents in different sectors (formal, informal, retirement and home) across years of education. Clearly, the informal sector attracts less-educated workers than its public and formal counterparts (and the public more so than the private).

In conclusion, a model studying the macroeconomic and occupational consequences of social security reforms should include the following features: (i) a well-paid and stable public sector with costly access, (ii) a public pension system that is relatively more attractive and expensive than the private pension system, and (iii) an informal sector absorbing low-skill individuals that are not participating the Social Security System. Although the model will be calibrated to Brazil, we think that these features are common across a large share of poor-to-medium income countries.

[10]Informality not only a feature of the Brazilian economy. Bacchetta et al. (2009) study the informal sector in developing countries, and provide evidence that informality is negatively correlated with GDP and GDP growth. Moreover, the study shows that the share of informal employment in total employment in the 2000s was 52% for Latin America, 78% in Asia and 56% in Africa. Bacchetta et al. (2009) state that the informal sector (excluding agriculture) accounted for 26% of GDP in Latin America in 2006. This study also shows that the informal sector attracts less educated people: on average, around 65% of all informal workers in Latin America are "low skill" workers.

[11]An informal worker is defined as an employee that does not have a signed working permit. We do not consider self-employed informal workers, as we will not model entrepreneurial activity. The share of informality in the labor force goes up to 40% if you include these type of agents, but we believe that modeling them is out of the scope of this paper.

[12]See Ulyssea (2018) for firm-level analysis of the informal sector.

3 The Model

The economic environment in this paper consists of a life-cycle model of occupational choice and retirement behavior. Individuals can be either in the private sector, working for the government, or retired from the labor force. All decisions are endogenous, so the individual will only retire from the labor force, or enter the public sector job if it is worth it.

We have a three-sector economy with public and private production, the latter being either formal or informal. The government is responsible for paying non-competitive wages to its workers in exchange for the production of a public good, and for managing a PAYG retirement system for both public and private sector retirees. In order to balance its budget the government taxes consumption, capital and labor income. The formal private firms use a Cobb-Douglas technology with capital and labor, whereas firms in the informal sector use a production function linear in labor. All firms act competitively and produce goods that are perfect substitutes in consumption.

Uncertainty in the economy comes from idiosyncratic shocks the private workers have in their labor efficiency, and the life span of the agents. Agents can save in a risk-free asset to smooth consumption against these shocks.

3.1 Demography, Preferences and Choices

The economy is populated by a continuum of mass one agents who may live at most J periods. Each agent has a time endowment of $\bar{H}$ hours per period.

There is uncertainty regarding the time of death in every period, so agents face a probability ψ_j of surviving to the age $j+1$ conditional on being alive at age j. The age profile of the population, denoted by $\{\varphi_j\}_{j=1}^J$, follows the law of motion $\varphi_{j+1} = \frac{\psi_j}{1+g_n}\varphi_j$, and satisfies $\sum_{j=1}^{J}\varphi_j = 1$, where g_n denotes the population growth rate[13]. The lifespan uncertainty entails that a fraction of the population leaves accidental bequests, which, for simplicity, are assumed to be equally distributed to all surviving individuals in a lump-sum basis (call it ζ). There are no private annuity markets in the economy.

Agents enjoy utility over effective consumption, $\tilde{c}_j$, and leisure time, l_j. They maximize lifetime expected utility:

$$\mathbb{E}_0\left[\sum_{j=1}^{J}\beta^j\left(\prod_{k=1}^{j}\psi_k\right)u(\tilde{c}_j, l_j)\right]$$

[13]The fraction of newborns is given by $\varphi_1 = \left[1+\sum_{j=1}^{J-1}(1+g_n)^{-j}\prod_{i=1}^{j}\psi_i\right]^{-1}$.

where β is the intertemporal discount factor, and $\mathbb{E}_j$ is the expectation operator conditional on age j. The period utility takes the form:

$$u(\tilde{c}_j, l_j) = \frac{[\tilde{c}_j^{\gamma} \cdot l_j^{1-\gamma}]^{1-\sigma}}{1-\sigma}$$

The effective consumption is given by $\tilde{c} = c + \epsilon Y_G$, with c being private consumption and Y_G the consumption of a public good. The parameter σ determines the risk aversion, γ denotes the share of consumption in the utility, and ϵ measures the relative importance of public consumption in overall consumption. All agents in the economy can save and lend their savings to a private competitive firm.

In this economy, agents can be private workers, public servants or retirees. Denote such individual state as $m \in \{P, G, R\}$, respectively. There is endogenous migration between these sectors, to be detailed in the next sections. Agents choose how much to consume, $c_j \geq 0$, and how much to work, h_j. We restrict the labor choice to be one element in $\{0, H < \bar{H}\}$. The remaining time is considered to be entirely leisure, so $l_j + h_j = \bar{H}$. We assume that public servants are obligated to go to work. In contrast, private workers and retirees choose between working or staying at home[14].

Agents can choose to take an open exam and try their luck into the public sector. Taking this exam is costly, where the time cost is a function of their current age, $c_{app}(j)$. We assume that retirees cannot enter the public sector, and that public workers cannot retake the exam.

As workers become older, and conditional on meeting the eligibility requirements, they can apply for social security benefits and become retirees next period within their respective sector. The informal private workers retire as private retirees, the same retirement sector as the formal private workers (there is no informal retirement sector).

The public sector and the retirement sector are absorbing states. Once a worker applies and enters the public sector, there is no turning back. The same is true for the application for social security benefits.

3.2 Labor Income, Efficiency, and Budget Constraints

Conditional on their respective sector, individuals make decisions on whether to work or not, and on asset accumulation. Let w_f denote the competitive wage paid by formal firms, and w_i the wage paid by informal firms. An individual aged j who decides to work $h_j \in \{0, H\}$ hours produces a

[14] We allow for retirees to endogenously choose whether to work or not. This is the case in Brazil, even though in some countries retirees are not allowed to work. We can easily shut down such market if we are interested in studying some other country's social security reforms, or introduce a punishment in terms of retirement benefit reduction in the case the retiree works.

total of units of consumption before taxes given by:

$$y_j(\omega, m) = \begin{cases} \omega \cdot e^{z+\eta_j} \cdot h_j & \text{if} \quad m = \textit{Private, Retirement} \\ (1+\theta) \cdot w_f \cdot e^{z_G+\eta_j} \cdot H & \text{if} \quad m = \textit{Government} \end{cases}$$

for $\omega \in \{w_f, w_i\}$.

We assume that the idiosyncratic productivity $\{z\}$ follows a first order Markov process with transition matrix Π. There is no uncertainty regarding the public sector, and z_G is the productivity that the private worker had when she decided to take the admission test for the public sector and succeeded. The function η_j is a deterministic age-specific component of labor efficiency. The parameter θ corresponds to the wage premium or economic rent that public sector workers receive relative to their counterparts in the formal private sector.

All agents in the economy pay capital income tax τ_k and consumption tax τ_c. Workers face labor income tax rate of $\tau_y(m)$, and those who are not retired must additionally contribute a fraction $\tau_{ss}(m)$ to the Social Security System. The revenue from $\tau_{ss}(m)$ is used to finance the social security benefits of the retirees, and the revenue from $\tau_y(m)$ finances overall government expenditures not related to the social security system. Retirees pay a tax rate of τ_b over their social security benefits. Informal workers do not pay labor income taxes, nor contribute to the SS system.

We assume that individuals save in a risk-free asset which pays a competitive interest rate r. They cannot have negative assets at any age, so that the amount of assets carried over from age j to $j+1$ is such that $a_{j+1} \geq 0$. Furthermore, given that there is no altruistic bequest motive and death is certain at age $J+1$, agents at age J consume all their assets, that is, $a_{J+1} = 0$. We normalize the continuation value after age J as zero.

The budget constraint for the non-retired individuals in the private sector is given by:

$$(1+\tau_c)c + a' = \begin{cases} [1+(1-\tau_k)r]a + (1-\tau_y(P))y_j(w_f, P) - \tau_{ss}(P)\min\{y_j(w_f, P), y_{\max}\} + \zeta & \text{if formal} \\ [1+(1-\tau_k)r]a + y_j(w_i, P) + \zeta & \text{if informal} \end{cases}$$

The budget constraint for the public sector workers is:

$$(1+\tau_c)c + a' = [1+(1-\tau_k)r]a + (1-\tau_y(G) - \tau_{ss}(G))y_j(w_f, G) + \zeta$$

Lastly, for the retirees receiving benefits b, the budget constraint is:

$$(1+\tau_c)c + a' = \begin{cases} [1+(1-\tau_k)r]a + (1-\tau_y(P))y_j(w_f, R) + (1-\tau_b)b + \zeta & \text{if formal} \\ [1+(1-\tau_k)r]a + y_j(w_i, R) + (1-\tau_b)b + \zeta & \text{if informal} \end{cases}$$

where b denotes the retirement benefits the retiree is entitled to. These benefits are calculated upon retirement, with rules that depend on the worker's state vector, to be discussed further in Section 4.

3.3 Public Sector Recruitment

We describe how the public sector recruitment works. In principle, one could incorporate different ways in which agents could migrate between the private and the government sectors. We take a stand and make this procedure as close as possible to the way it happens in Brazil.

According to constitutional rules, the hiring process of civil servants in Brazil is given by public competition. Candidates must take a nationwide open exam, and only those who scored the best fill up the job vacancies. In the model, agents who want to work in the public sector must perform well at costly "exams", and only those who obtain the best grades become eligible to fill a predetermined number of job positions. Once a private worker takes the test and succeeds, she becomes a public servant next period, and must work for the government until retirement.

The timing is the following. First, an agent chooses to apply, at age j, paying the time cost $c_{app}(j)$. Her score is revealed in the next period: $q \sim U[0,1]$. If $q \geq \bar{q}$, she will necessarily work for government from age $j+1$ onward. Otherwise, she will remain a private sector agent. The threshold score, $\bar{q}$, is chosen by the government in equilibrium to balance the demand and supply of public servants.

3.4 Social Security

Private and public workers have specific social security systems with different rules. Workers in the private sector have two retirement modalities, contribution and age. In the first scheme the worker may apply for retirement after contributing for a number of years (35, in the case of Brazil) even if she has not reached the normal retirement age. In the age modality, similar to the rest of the world, a worker may apply for retirement after reaching the normal retirement age (65 years old in the case of Brazil) and having contributed for a minimum number of years. In the model, the agent will choose optimally the retirement age and the modality.

The value of the retirement benefits, b, is calculated as a fraction of average past earnings, x, that will depend on the age of retirement, the modality, the number of years of contribution, among other institutional factors to be detailed in Section 4. The law of motion for x is given by:

$$x_{j+1} = \frac{x_j(j-1) + min\{y_j(w_f, P), y_{\max}\}}{j}, \quad \text{for } j = 1, 2, ..., j^r,$$

where $y_{\max}$ is the maximum taxable income, or social security ceiling.

Public workers also have two retirement modalities. If she is older than a certain age (60 years old in the case of Brazil) and contributed for a minimum number of years she can retire under the contribution modality and the benefits will be in essence equal to her current labor income. Civil servants older than 65 can retire under the age modality. In this case, however, individuals are entitled only to a proportion of their last wage depending on the number of years of contribution.

3.5 Value Functions

We now detail the agents' problems. We first describe the state space and the policy functions, then we detail the value functions for each agent in the economy.

We divide an individual state depending on what sector of the economy she is located. The state of an agent in the private sector is $s_P = (j, a, z, x, t_c) \in S_P \equiv \{1, ..., J\} \times \mathbb{R}_+ \times \mathcal{Z} \times \mathcal{X} \times \{0, ..., J\}$, where j is her age, a her asset holdings, z is the agent's idiosyncratic productivity, x is her average past earnings, and t_c is the number of years contributed to the SS system. The state of a public worker is $s_G = (j, a, z, t_c, t_G) \in S_G \equiv \{1, ..., J\} \times \mathbb{R}_+ \times \mathcal{Z} \times \{0, ..., J\} \times \{0, ..., J\}$, where t_G is the number of years worked in the government. As for the retirees, their relevant state is given by $s_R = (j, a, z, b) \in S_R \equiv \{1, ..., J\} \times \mathbb{R}_+ \times \mathcal{Z} \times \mathcal{B}$, where b denotes the retirement benefits. The state variables are obtained based on the model specification above, as well as the determinants of the retirement benefits in each sector (x,t_c, and t_G), further detailed in Section 4.

Solving the recursive problem yields the policy functions for working hours $d^h(s_m) \in \{0, H\}$, asset holdings $d^a(s_m) \in \mathbb{R}_+$ and consumption $d^c(s_m) \in \mathbb{R}_{++}$ for all $m \in \{P, G, R\}$; retirement $d^{ss}(s_m) \in \{0, 1\}$ for $m \in \{P, G\}$; informality $d^{inf}(s_m) \in \{0, 1\}$ for $m \in \{P, R\}$, and the public sector application $d^{app}(s_P) \in \{0, 1\}$.

3.5.1 Retired Workers

A retiree chooses between working formally, informally, or staying at home. Her value function is given by:

$$V(s_R) = \max_{\substack{(c,a')\geq 0,\, h\in\{0,H\} \\ inf\in\{0,1\}}} u(\tilde{c}, \bar{H} - h) + \beta\psi_j \cdot \mathbb{E}\left[V(s'_R)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = \begin{cases} [1 + (1-\tau_k)r]a + (1-\tau_y(P))y_j(w_f, R) + (1-\tau_b)b + \zeta, & inf = 0 \\ [1 + (1-\tau_k)r]a + y_j(w_i, R) + (1-\tau_b)b + \zeta, & inf = 1 \end{cases}$$

where the evolution of the state follows $s'_R = (j+1, a', z', b)$, and $\mathbb{E}\left[V(s'_R)\right] = \sum_{z'} \Pi(z, z')V(s'_R)$ is the standard expected value conditional on the current productivity, z.

3.5.2 Public Servants

At each age j, a public sector worker decides whether to retire from the labor force or not. Letting ret be an indicator function that assumes the value of 1 if the individual decides to retire and the zero otherwise, the value function of the public employee is given by:

$$V(s_G) = \max_{(c,a')\geq 0,\, ret\in\{0,1\}} u(\tilde{c}, \bar{H} - H) + \beta\psi_j \cdot \{(1 - ret) \cdot V(s'_G) + ret \cdot V(s'_R)\}$$

$$\text{s.t.} \quad (1 + \tau_c)c + a' = [1 + (1 - \tau_k)r]a + (1 - \tau_y(G) - \tau_{ss}(G))y_j(w_f, G) + \zeta$$

with next period state being $s'_G = (j + 1, a', z, t_c + 1, t_G + 1)$ when she does not retire and $s'_R = (j + 1, a', z, b' = b(s_G))$ in case of retirement. The variable $b(s_G)$ denotes the retirement benefits the agent will receive as a function of her state variables[15].

3.5.3 Private Workers

At each age j, the formal private worker makes decisions in three dimensions, in addition to the consumption/saving choice. First, if eligible, she decides whether to retire from the labor force or not. Second, she decides whether to work (formally or informally) or to stay at home. Third, she decides if she takes the public exam. Letting app be an indicator function that assumes the value of 1 if the individual decides to take the public exam and the zero otherwise, the private worker value function can be written as[16]:

$$V(s_P) = \max_{\substack{(c,a')\geq 0,\, h\in\{0,H\} \\ (ret,app,inf)\in\{0,1\}^3}} u(\tilde{c}, l) + \beta\psi_j \cdot \left\{ \begin{aligned} & ret && && \cdot \mathbb{E}[V(s'_R)] \\ & + (1 - ret) \cdot && app \cdot (1 - \bar{q}) \cdot && V(s'_G) \\ & + (1 - ret) \cdot && app \cdot \quad \bar{q} && \cdot \mathbb{E}[V(s'_P)] \\ & + (1 - ret) \cdot (1 - app) && && \cdot \mathbb{E}[V(s'_P)] \end{aligned} \right\}$$

$$\text{s.t.} \quad (1+\tau_c)c+a' = \begin{cases} [1 + (1 - \tau_k)r]a + (1 - \tau_y(P))y_j(w_f, P) - \tau_{ss}(P)\min\{y_j(w_f, P), y_{\max}\} + \zeta, & inf = 0 \\ [1 + (1 - \tau_k)r]a + y_j(w_i, P) + \zeta, & inf = 1 \end{cases}$$

$$(x', t'_c) = \begin{cases} \left(\frac{(j-1)x+\min\{y_j(w_f,P),y_{max}\}}{j}, t_c + 1\right) & \text{if } inf = 0 \text{ and } h = H \\ (x, t_c) & \text{otherwise} \end{cases}$$

[15]The benefits function $b(\cdot)$ for both private and public workers are specified in Section 4.

[16]Because retirement is a deterministic choice and we do not allow retirees to work in the government, there is no continuation value where the agent retires from the labor force and takes the public exam.

$$l + h + app \cdot c_{app}(j) = \bar{H}$$

The first term of the continuation value inside the brackets corresponds to retirement, the second to not retiring, applying to a public job and passing the exam, the third to not passing the exam (and not retiring), and the fourth and final term corresponds to the decision of not retiring and not applying to a public job.

The evolution of the state variable is, for the case of retirement, $s'_R = (j+1, a', z', b' = b(s_P))$; for the case of entering the public career, $s'_G = (j+1, a', z, t'_c, 1)$; and, finally, $s'_P = (j+1, a', z', x', t'_c)$ for the case where she continues a private sector agent.

3.6 Agents' Stationary Distribution

The stationary distribution of agents is characterized by probability distribution functions $\mu_m : S_m \to [0,1]$, for all $m = \{P, G, R\}$, such that $\sum_{(m,s_m)|j} \mu_m(s_m) = \varphi_j$ for all $j = \{1, ..., J\}$. That is, $\mu_m(s_m)$ is the measure of individuals in sector m and state s_m in the population.

In equilibrium, the stationary distribution of agents is constructed by forward induction using the policy functions derived in the previous section. However, we have to take a stance on the distribution of agents entering the economy (i.e., such that $j = 1$). We assume that: (i) every agent starts her life-cycle with zero initial assets, zero average past earnings and zero time of contribution; (ii) everybody starts as a worker in the formal private sector; and (iii) the initial distribution of the idiosyncratic productivity is the invariant distribution of the Markov process for $\{z\}$.

For each of the remaining ages, the distributions is derived using forward induction, considering the agents' policy functions, the transition matrix for the income process, the survival probabilities, and the probability of succeeding in the public exam and entering the public sector. A formal derivation of the equilibrium distribution can be found in Appendix A.

3.7 Technology

We assume that there are two representative firms producing perfect substitute goods. One operates in the formal sector and one in the informal sector. The first one produces using capital and labor, whereas the second one uses only labor. Both of them act competitively and maximize profits given input prices.

The production function of the formal sector is Cobb-Douglas: $Y_f = A_f K^{\alpha} N_f^{1-\alpha}$, where K and N_f are the aggregate capital and private labor inputs, α is the capital share and A_f denotes the formal sector total factor productivity (TFP). Capital is assumed to depreciate at a rate δ each period. The problem of the formal firm is:

$$\max_{K,N_f} A_f K^\alpha N_f^{1-\alpha} - w_f N_f - (r+\delta)K$$

Informal firms have linear technology in labor: $Y_i = A_i N_i$ and maximize profits according to:

$$\max_{N_i} A_i N_i - w_i N_i$$

As the focus of our analysis is on the household-level responses to social security reforms, we abstract from firm heterogeneity within the formal and informal sectors. By doing so, we deviate from the literature that looks at the impact of informality on development and firm-level behavior[17]. That literature typically considers an increasing, strictly convex labor costs for operating under informality, which represents the probability of being caught by government authorities. Our simplified assumption, together with our calibration exercise, implicitly embeds these costs on the informal sector productivity parameter, A_i.

3.8 The Government Sector

The government taxes consumption, capital, income, and social security benefits to finance the social security coverage, the payroll of public servants and its own, non-productive consumption. We assume that the government consumes a constant fraction of the formal GDP: $C_g = \alpha_g Y_f$.

In the labor market, the government hires a share $\bar{N}_G \in [0,1]$ of the population as public servants, and uses them to produce a public good Y_G. We assume that the government production function is linear in the effective labor supply: $Y_G = L_G$.

In equilibrium, the government is responsible to choose $\bar{q}$ in order to balance the demand and supply of public workers. This assumption hinges on the fact that, over the 2005-2013 period, the Brazilian government employed consistently around 5% of the population[18].

3.9 Equilibrium

We now define the *recursive competitive equilibrium* in this economy. A *recursive competitive equilibrium* consists of allocations of households and firms, prices (wages and interest rate), government taxes and threshold score, a stationary distributions of agents, bequests and public goods such that:

[17] See Ulyssea (2017), Meghir et al. (2015), Almeida and Carneiro (2012) and de Paula and Scheinkman (2011).

[18] This figure was calculated using PNAD data from 2005-2013. The figure of the fraction of statutory employees in population over time is available upon request.

(i) households and firms optimize; (ii) individual and aggregate behaviors are consistent; (iii) the government sets threshold scores and consumption taxes to balance the size of the public sector in the population as well as its budget constraint; (iv) the stationary distributions evolve according to the policy functions of the agents; and (v) the amount of public good and the amount of bequests are consistent with individual behavior. A complete equilibrium definition can be found in Appendix B, and the algorithm used to compute the equilibrium is detailed in Appendix C.

4 Data and Calibration

We now describe the calibration exercise. Using publicly available micro and macro data from different sources, we calibrate the model to match features of the Brazilian economy in 2013. Tables 5 and 6, by the end of this section, summarize the parameters values of this exercise.

4.1 Data Sources

At the aggregate level, we use data from the 2013 Brazilian National Accounts to calculate the fraction of GDP consumed by the government, the 2013 Social Security Annual Report to calculate the social security deficit and statistics on public versus private retirement, and the 2013 IBGE mortality tables to obtain the survival probabilities.

At the micro level, we use two data sets, both provided by the IBGE. The first is the 2013 PNAD, a cross-section household survey that is representative at the national level. We use this data set to calculate the distribution of agents across the three main sectors in the model (private, public and retirees) as well as the distributions of economic participation, test takers, and statistics on retirees by age.

The second data set, used for the estimation of the income process, the age-efficiency profile and the public sector wage premium, is the 2012-2018 PNADC. This survey is a quarterly rotating panel with information on labor market outcomes for a nationally representative sample. Its main goal is to produce indicators that monitor quarterly fluctuations in the workforce. It follows a rotation scheme where each household is visited five times during five consecutive quarters, allowing us to construct a panel of individuals in an interval of one year.

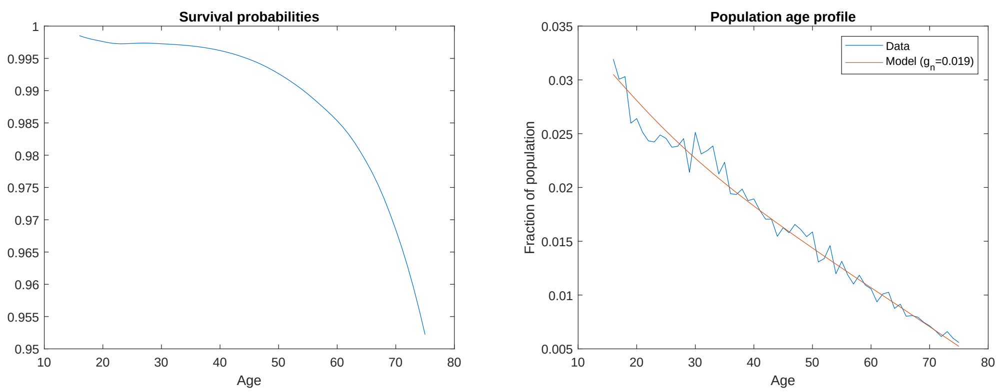
Even though they are similar, the PNAD has better information on both demographics and individuals out of the labor force, as the main purpose of the PNADC is to generate labor force statistics. In both data sets we focus the analysis on men between 16 and 75 years old. We define an informal worker as a worker that does not have a legal working contract (*carteira de trabalho*), hence does not participate the tax system and is not covered by the Social Security Administration.

A government agent is a statutory government employee, as these are subject to the segregated retirement scheme[19].

4.2 Demography and Endowments

A model period corresponds to one year, and we assume agents live from 16 to 75 years old, so $J = 60$. The survival probabilities are taken directly from the IBGE's 2013 mortality tables. We set the population growth rate at $g_n = 0.019$ to match the population age profile obtained from the 2013 PNAD. Figure 7 plots the survival probabilities as they are used in the model (left panel) and a comparison between the population age profile in the data and in the model (right panel).

Figure 7: Calibrated survival probabilities and age profile



Notes: This figure shows the survival probabilities (left) and the population age profile (right), comparing the model-implied profile (red) with the data (blue). Sources: 2013 IBGE Mortality Tables and 2013 PNAD.

We assume agents have $\bar{H} = 8,760$ hours per year, and that they can either work zero or 8 hours per day (the contractual full-time routine in Brazil in 2013), which implies $H = 2,016$ hours.

4.3 Preferences and Technology

First we detail the calibration of the preference parameters. The value of β is chosen so that the capital-to-output ratio is 2.5. This value is obtained from Morandi (2016), which applies the Perpetual Inventory Method to compute a historical series of the capital stock in Brazil, and lies in the range of 2.5 and 3, values commonly used in the Macro literature for Brazil[20]. We set the risk

[19] Politicians, Central Bank workers and administrative/clerical jobs in the government are examples of statutory workers. Employees of publicly owned enterprises are not considered statutory public servants, and are subject to the private social security scheme.

[20] See dos Santos and Cavalcanti (2015), Glomm et al. (2009) and Ferreira and do Nascimento (2005).

aversion parameter at 2.5 based on the literature on consumption, surveyed by Attanasio (1999). This value is in line with estimates of the risk aversion parameter for the Brazilian economy[21]. The consumption share in the utility, γ, is chosen to match the participation rate of 72.3% in 2013, according to the PNAD. Lastly, we set ϵ to 0.5 following the work of Ferreira and do Nascimento (2005).

The technology parameters were calibrated as follows. For the formal sector, we set the capital share in output as $\alpha = 0.4$, and the depreciation rate at $\delta = 6\%$, as commonly used in the Macro literature[22]. We calibrate the formal sector TFP, A_f, to match the 2013 GDP per capita of R$26,520. As for the informal sector, we calibrate the informal firms' productivity of labor, A_i, to match a share of informal workers in the population of 18.9%.

4.4 Estimation of Labor Income

We now turn to the estimation of the the stochastic process for $\{z\}$, the age-efficiency profile, $\{\eta_j\}$, and the public sector wage premium, θ. As mentioned above, we use micro data from the IBGE's *Pesquisa Nacional por Amostra de Domicílios Contínua* (PNADC) from 2012q1 to 2018q4.

We assume that the age-efficiency profile is quadratic: $\eta_j = \alpha_1^\eta j + \alpha_2^\eta j^2$, and use the model to write down the log hourly wages of individual i as:

$$\ln\left(\frac{y}{h}\right)_{ij} = C + \alpha_1^\eta j + \alpha_2^\eta j^2 + z_{ij} \tag{1}$$

where C varies only with the working sector of the agent.

We estimate the state space, $\mathcal{Z}$, and the transition probabilities, $\Pi(z, z')$, of the idiosyncratic labor productivity non-parametrically[23], following Hansen et al. (2014), De Nardi et al. (2016), and Ferreira and Gomes (2017). Because labor income fluctuation in the model only happens in the private sector, we restrict the data to this sector. First, we obtain the empirical counterparts of z_{ij} as the residuals from a regression of log hourly wages on age and age-squared. We then split these residuals into five groups: the top 5%, the next 20%, 25%, 25%, and the bottom 25%, and set each element in the state space $\mathcal{Z}$ as the mean within each respective group. The construction of the transition probabilities explores the panel dimension of the data. We calculate the fraction of individuals that migrated between every two groups over a year.

[21]See Gandelman and Hernandez-Murillo (2015) and Fajardo et al. (2012).

[22]See Parente and Prescott (2002) for evidence on the depreciation rate and Gomes et al. (2005) for evidence on the capital share of formal output.

[23]We believe a non-parametric estimation of the income process is important as it captures empirical deviations from the normality assumption highlighted, for the Brazilian case, by Gomes et al. (2019). Guvenen et al. (2015) and De Nardi et al. (2016) also find large deviations from normality in the U.S. labor market.

Table 3: Idiosyncratic productivity process

Transition matrix: $\Pi(z, z')$	z_1	z_2	z_3	z_4	z_5
z_1	0.659	0.220	0.089	0.029	0.003
z_2	0.220	0.473	0.233	0.070	0.004
z_3	0.082	0.246	0.479	0.183	0.009
z_4	0.038	0.090	0.248	0.573	0.051
z_5	0.022	0.030	0.055	0.252	0.642
State space: $\mathcal{Z}$	-0.715	-0.249	0.074	0.548	1.542

Notes: This table shows the results for the estimation of the idiosyncratic labor income process. The first 6 rows display transition probabilities between every two periods. Rows represent productivities at t and columns represent productivities at $t+1$. The last row shows the elements in the discretized state space.

The regression results are displayed in the first column of Table 4, and the estimated grid and transition probabilities are shown in Table 3. There is asymmetry in both the transition probabilities as well as the elements in the state space (around zero). This is fundamentally different than the symmetry imposed by traditional methods for estimating the income process, such as Tauchen (1986), and we believe is a better representation of the data.

The estimation of the public sector wage premium and the age-efficiency profile also rely on estimating Equation (1). The Markovian nature of the idiosyncratic productivity process allows us to write:

$$\ln\left(\frac{y}{h}\right)_{ij} = C + \alpha_1^\eta j + \alpha_2^\eta j^2 + \underbrace{g(z_{ij-1}) + \varepsilon_{ij}}_{=z_{ij}}$$

The $g(z_{ij-1})$ term above shows the potential omitted variable problem Regression (1) faces. To deal with this problem, we include a rich set of controls (such as education, occupation and sector), and individual fixed effects. A public sector wage premium of 9.8% ($= \exp(0.093) - 1$) is estimated from the "Public servant" coefficient in Column (3) of Table 4. To avoid issues from selection of older people, we restrict our sample to 25-55 years old individuals and estimate the age-efficiency profile coefficients of $\alpha_1^\eta = 0.028617$ and $\alpha_2^\eta = -0.000313$, as shown in Column (4) of Table 4. Figure 8 shows the estimated age-efficiency profile.

Table 4 also highlights the importance of the panel dimension in the data. Column (2) calculates the public sector wage premium without controlling for individual fixed effects, and it finds a coefficient value of 17%, almost two times larger than the estimation of column (3). This clearly reflects the selection on fixed individual characteristics that drive high-type agents into the public sector, as discussed in Section 2.

Figure 8: Estimated age-efficiency profile

Notes: This figure plots the age efficiency profile, $\eta_j = \hat{\alpha}_1^\eta j + \hat{\alpha}_2^\eta j^2$, implied by the age coefficients estimated in column (4), Table 4. Sources: 2012-2018 PNADC.

Table 4: Estimation of the income process variables

	(1)	(2)	(3)	(4)
Public servant		0.169*	0.094*	0.090*
		(0.004)	(0.005)	(0.006)
Informal worker		-0.188*	-0.056*	-0.053*
		(0.002)	(0.002)	(0.003)
Age	0.061*	0.032*	0.031*	0.029*
	(0.000)	(0.000)	(0.001)	(0.003)
Age^2	-0.00063*	-0.00028*	-0.00033*	-0.00031*
	(0.000)	(0.000)	(0.000)	(0.000)
Individual FE			✓	✓
Controls		✓	✓	✓
Time FE	✓	✓	✓	✓
Sample	Private	All	All	25-55
N	1,832,912	2,053,458	1,773,520	1,270,749
R^2	0.196	0.615	0.899	0.898

Notes: This table shows the main coefficients of Regression (1). We use the PNADC data from 2012q1-2018q4, focusing on men between 16 and 75 years old. Column (1) focus on individuals in the private sector, columns (2) and (3) consider the whole sample, and column (4) restricts the sample to 25-55 years old individuals. Control variables are: tenure, race (when there are no individual fixed effects), and occupation, sector, and education fixed effects. Robust standard errors in parentheses. * $p < 0.001$.

4.5 Social Security System

The social security in Brazil is a pay-as-you-go system, which transfers income from workers to retirees. The system is financed with payroll taxes, and has two very different regimes - the private sector regime and the public sector regime. The benchmark year of our calibration is 2013, hence the retirement benefits structure is modeled in order to mimic the retirement rules that used to prevail in Brazil at that year, before the implementation of a major social security reform, objective of analysis in the counterfactual sections.

4.5.1 Private Sector

The private sector regime is organized under the INSS, which stands for *Instituto Nacional do Seguro Social* (National Institute of Social Security). Under the INSS retirement sector, there are two modalities of retirement - the age modality and the contribution modality.

We first detail the eligibility requirements. If the worker is older than the normal retirement age, which is 65 years old, and has contributed to the retirement system for more than 15 years, she can apply for retirement under age modality. If the worker has not achieved the normal retirement age, but have contributed for more than 35 years to the social security system, she can retire under the contribution modality.

In both modalities, the value of the benefits is calculated as a fraction of average past earnings, x:

$$b(j^r, x, mod, t_c) = \Psi(j^r, mod, t_c) \cdot x$$

where $mod \in \{Contrib, Age\}$ stands respectively for the contribution and the age modalities of retirement, $\Psi(j^r, mod, t_c)$ denotes the retirement replacement rate as a function of the age in which the worker retires from the labor force, j^r, the retirement modality, mod, and the number of years that the worker contributed formally to the social security system, t_c.

The average lifetime earnings, x, is calculated by taking into account individual earnings up to the age of withdrawal from the labor force that are lower than the maximum taxable income, $y_{\max}$. Thus, the law of motion for x can be written as:

$$x_{j+1} = \frac{x_j(j-1) + min\{y_j(w_f, P), y_{\max}\}}{j}, \quad \text{for } j = 1, 2, ..., j^r \tag{2}$$

Only earnings from the formal sector are considered in the calculation of x, as we assume that individuals in the informal sector do not contribute to the social security system[24].

For those who retire under the contribution modality, the replacement rate is given by:

$$\Psi(j^r, Contrib, t_c) = f(j^r, t_c)$$

[24]Even though informal workers can contribute so the SS system, only a small fraction do so. In 2013, nearly 84% of the informal workers have not contributed to the SS system. On average from 2002-2013, only 11% of the informal workers contributed.

where $f(t^r, t_C)$ is commonly known as the *fator previdenciário* (social security factor). Such discount was implemented by the Fernando Henrique Cardoso's presidency, in order to discourage the early retirement that occurred in Brazil. Its formula is given by:

$$f(j^r, t_c) = \frac{0.31t_c}{E(j^r)}\left[1 + \frac{(j^r + 0.31t_c)}{100}\right]$$

where $E(j^r)$ is the life expectancy of the individual at the retirement age j^r, and t_c is the number of years social security contributions. Depending on the number of years the worker has contributed to the social security system, and on the age of retirement, the social security factor can be greater than 1[25].

Under the age modality, the worker can choose between the social security factor rule or an alternative replacement rate, which starts at[26] 85% of average earnings and can increase up to 100%. Hence, the replacement rate for the age modality reads:

$$\Psi(j^r, Age, t_c) = \max\{f(j^r, t_c), \tilde{\Psi}(t_c)\}, \quad \tilde{\Psi}(t_c) = \min\left\{0.70 + \frac{t_c}{100}, 1\right\}$$

Lastly, we calibrate the contributions to the private social security system following the 2013 social security rules. Private workers paid 8%, 9% or 11% of their labor income up to the social security ceiling y_{max} in the following manner:

$$\tau_{ss}(P) = \begin{cases} 8\% & \text{if } 0 \leq \min(y, y_{\max}) \leq R\$\,14,972 \\ 9\% & \text{if } R\$\,14,972 < \min(y, y_{\max}) \leq R\$\,24,954 \\ 11\% & \text{if } R\$\,24,954 < \min(y, y_{\max}) \end{cases} \tag{3}$$

and private retirees have no tax on their benefits. We set the private social security ceiling $y_{max} = R\$\,10,000$ to match the private social security deficit as a fraction of GDP of 0.94%.

4.5.2 Public Benefits

According to the Brazilian Constitution public servants have rights to a different pension system. This pension system is also split into two modalities, a contribution and an age modality.

The retirement modality determines the replacement rate applied to the wage base, determining the public retiree's benefits. Differently from the private retirement sector, where the wage base is a bounded average of past earnings, the wage base in the public retirement sector, until 2013, did

[25] For instance, a 58 years old worker who contributed for 35 years receives only 80% of her past earnings upon retirement. However, if the same worker contributed for 45 years, her replacement rate would be 106%.

[26] This number comes from the 70% in the formula for $\tilde{\Psi}(t_c)$ plus the 15% required for eligibility.

not have an upper limit and corresponded to the average of the 80% highest wages received during the public career. Since we assume that the productivity of a worker, after entering in the public sector, does not change this average equals the last wage.

Two other differences relative to the private retirement scheme are that retirement in the public sector is mandatory at age 70, and the individual must have at least 10 years working as a public servant to be eligible for retirement.

Civil servants older than 60 years old that contributed for at least 30 years can retire under the contribution modality. The benefits match the worker's current labor income, y:

$$b(y, Contribution) = y$$

Civil servants older than 65 can retire under the age modality. In this case, individuals are entitled to a proportion of their last wage:

$$b(y, t_c, Age) = \min\{\frac{t_c}{35}, 1\} \cdot y$$

Finally, following the 2013 tax code, we assume that public workers compulsorily pay 11% of their income to the social security system, and that public retirees pay an 11% tax over the benefits they have in excess to the private social security ceiling.

4.6 Government Sector

The labor and capital tax rates are chosen based on the Brazilian macro literaturel[27]. We set them at $\tau_y(P) = 18\%$ and $\tau_k = 15.5\%$. Following Immervoll et al. (2006), we calibrate the labor income tax of the public servants to $\tau_y(G) = \frac{\tau_y(P)}{2} = 9\%$. The consumption tax rate is chosen to balance the government budget constraint in equilibrium. Lastly, we use the 2013 Brazilian National Accounts to calculate that government consumption accounts for $\alpha_g = 19\%$ of GDP.

We use the PNAD data to calibrate the variables associated with the government sector. We assume that the time cost for taking the public exam is quadratic: $c_{app}(j) = \alpha_2^{app} j^2 + \alpha_1^{app} j + \alpha_0^{app}$, and calibrate the parameters to minimize the distance between the fraction of test takers by each age in the model and in the data. This procedure implies a convex cost function with coefficients[28] $\alpha_0^{app} = 800$, $\alpha_1^{app} = -90$, and $\alpha_2^{app} = 2.7$. The size of the public sector is calculated directly from the PNAD data: $\bar{N}_G = 5\%$.

[27] See, among many, Glomm et al. (2009), and Pereira and Ferreira (2010) .

[28] A plot of the cost function in annual hours as a function of age is available upon request.

4.7 Calibration Results

Tables 5 and 6 summarize the external and internal calibration procedures, respectively:

Table 5: External calibration

Parameter	Description	Value	Source
J	Maximum age	60	Agents live from 16-75
$\{\psi_j\}$	Survival probabilities	Figure 7	IBGE
g_n	Population growth rate	1.9%	IBGE
$\bar{H}$	Time endowment	8,760 hours	24 hours/day × 365 days/year
H	Working hours	2,016 hours	8 hours/day × 252 weeks/year
σ	Risk aversion	2.5	Attanasio (1999)
ϵ	Public good coef.	0.5	Ferreira and do Nascimento (2005)
α	Capital share in output	0.4	Standard value
δ	Depreciation rate	0.06	Parente and Prescott (2002)
$\mathcal{Z}$	Productivity space	Table 3	PNADC
$\Pi(z, z')$	Transition probabilities	Table 3	PNADC
$\{\eta_j\}$	Age-efficiency profile	Figure 8	PNADC
θ	Public sector wage premium	9.8%	PNADC
$b(\cdot)$	Benefits function	Section 4.5	2013 rules
$\tau_y(P)$	Private sector's income tax	18%	Literature
$\tau_y(G)$	Public sector's income tax	9%	Immervoll et al. (2006)
τ_k	Capital tax	15.5%	Literature
$\tau_{ss}(G)$	G worker SS contribution	11%	2013 tax rates
$\tau_{ss}(P)$	P worker SS contribution	Equation (3)	2013 tax rates
τ_b	G retiree benefit tax	11%	2013 tax rates
α_g	$\frac{\text{Govt Consumpt.}}{\text{GDP}}$	18.9%	National Accounts
$\bar{N}_G$	Size of the government	5%	2005-2013 PNAD

Notes: This table summarizes the externally calibrated parameters, discussed throughout Section 4.

Table 6: Internal calibration

Parameter	Target	Model	Data
$\beta = 0.949$	K/Y	2.6	2.5
$\gamma = 0.246$	Participation Rate	71.7%	72.3%
$A_f = 2.05$	GDP per capita	R$ 25,945	R$ 26,520
$A_i = 3.06$	Informal sector size	19.1%	18.9%
$\alpha_0^{app} = 800$			
$\alpha_1^{app} = -90$	Test takers age profile	Figure 10	
$\alpha_2^{app} = 2.7$			
$y_{max} = R\$\,10,000$	Private SS deficit/GDP	1.07%	0.94%

Notes: This table shows the internal calibration results. The numerical implementation of the model is discussed in Appendix C.

5 Equilibrium Features and External Validation

This section details the quantitative features of the calibrated model. The model generates a distribution of agents across the three main sectors (private, public and out of labor force), a distribution of endogenous choices (public sector application and informality), and other macroeconomic aggregates (e.g., overall social security deficit and early retirement) that match the data closely.

We first discuss the main equilibrium variables in the calibrated economy. The interest rate is 8.5%. This value is relatively high by international standards, but not for Brazil. The formal wage rate (3.9 Reais per hour) is 1.3 times larger than the informal wage. As discussed in the motivation section, this discrepancy in wage rates reproduces the fact that the informal sector absorbs low-productivity individuals. Conditional on taking the public exam, an individual has a 22% chance of getting the public job[29]. Finally, the consumption tax rate is 20.7%. The simplified tax structure used in the model yields an equilibrium consumption tax rate which is close to what is observed in the Brazilian data[30].

How are agents distributed across the private, public and retirement sectors? Even though we target a government that absorbs 5% of the population, the model reproduces a private sector that accounts for 84%, and a retirement sector that accounts for the remaining 11% of the population. In the data, these numbers are 82% and 13%, respectively. Breaking down the agents' distributions by age, Figure 9 shows that our calibrated model matches the age profile of individuals across sectors well. It captures relevant features of the data such as the age profile of public servant, which is key in determining the public sector social security deficit. It also reproduces part of the early retirement in the private sector, which determines the extend to which minimum retirement age provisions are effective in reducing the private sector social security deficit.

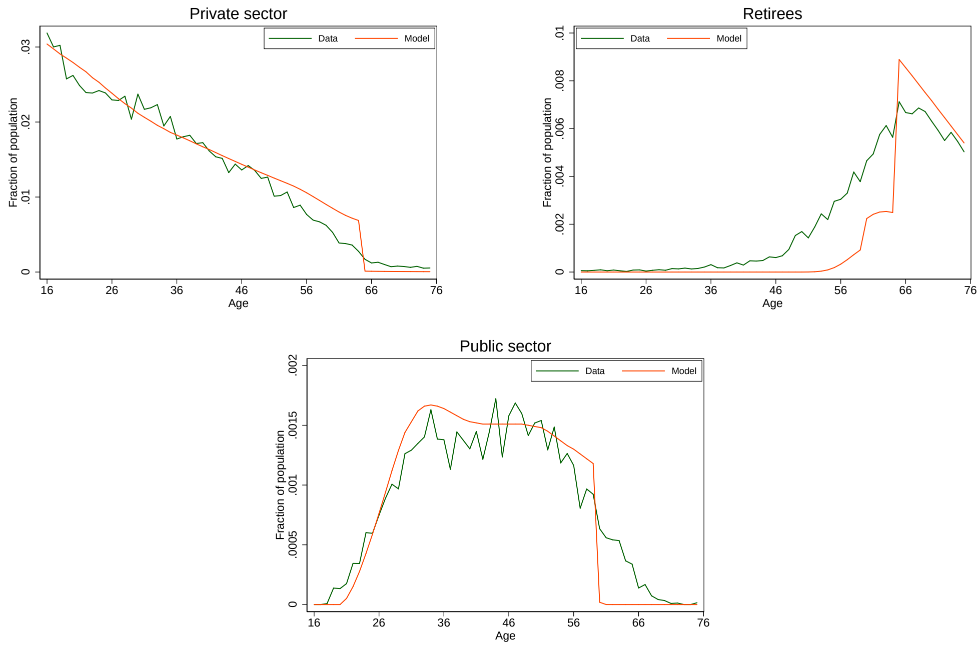
Even though the calibration procedure targets the overall participation rate, the model reproduces the labor force participation by age quite well, as shown in the top-left panel of Figure 10. However, it overestimates the number of young people in the labor market. Young agents in the model economy have low asset levels, and have to work early in life to compensate this lack of resources. Moreover, we do not consider human capital accumulation, nor schooling decisions. Most individuals between 16-18 years old are probably finishing their studies, preparing to enter the labor market, and still living with their parents, features that are not present in the model.

The timing of the public sector application decision determines the duration of the careers in the sector. This, in turn, will govern the amount of social security taxation and spending related to public servants. Despite the fact that we model the public sector application decision quite parsi-

[29]Unfortunately we do not have data on the public sector qualification exams to validate this number. However, by comparing the age profile of applicants in the top-right panel of Figure 10, we are confident that our model captures the public sector application decision in the data fairly well.

[30]For instance, see dos Santos and Pereira (2010).

Figure 9: Agents' equilibrium distribution by age and sector



Notes: This figure plots, for each age, the percentage of the population allocated in each sector.

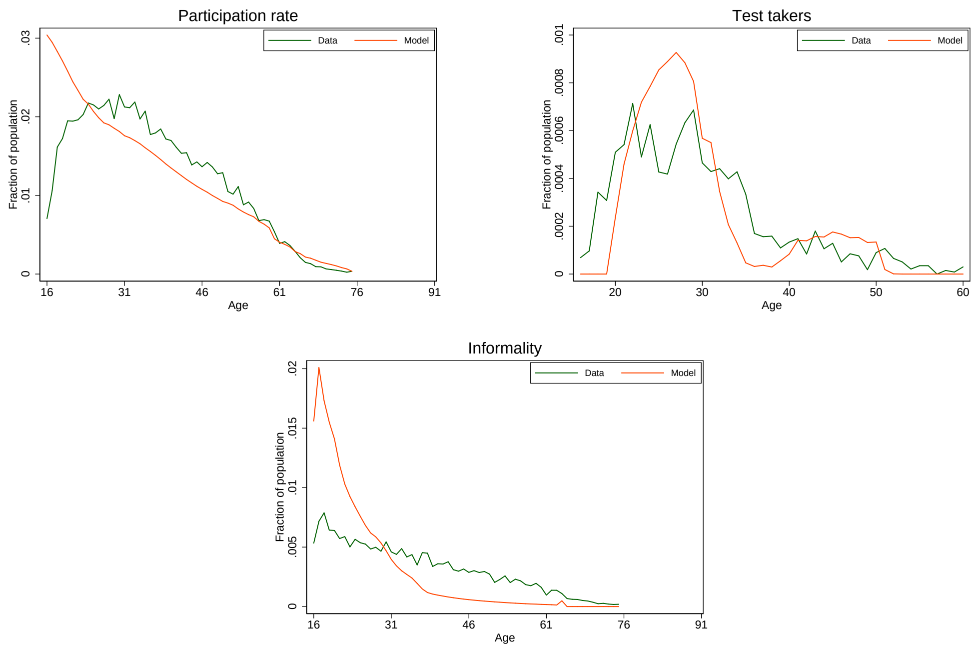
moniously, Figure 10 shows that the calibrated economy generates a life cycle profile of applicants that follows closely to the data. The nonlinearities over the life cycle are well captured by a convex application cost function.

The bottom panel of Figure 10 plots the size of the informal sector by age. In the model, informality decreases significantly as individuals become older, a pattern that is also observed in the data, even though the intensity of the reduction is very different. This is because, from the agent's point of view, it is too easy to alternate between formal and informal sector, whereas in reality this transition should be more sluggish. Moreover, as discussed above, the model overestimates the share of young agents in the labor force and given their relative low productivity and experience, they tend to work in this sector in greater proportion than what is observed in the data.

Figure 11 plots the distribution of the idiosyncratic labor productivity z across the three sectors. In a stylized manner, it reproduces some of the features observed in the data (Figure 6). There is a high concentration of low skill workers in the informal sector; in contrast, the distribution of z in the public sector is more concentrated in high skill individuals; and, finally, in the formal sector skills tend to be more evenly distributed.

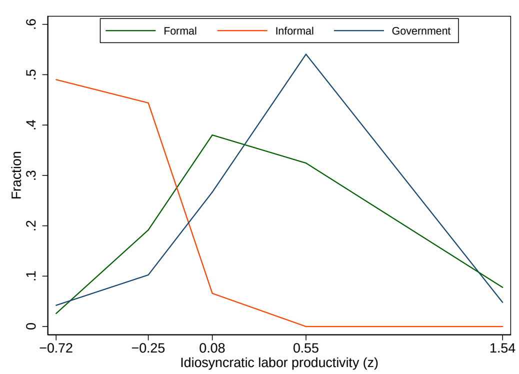
In the model economy these discrepancies of idiosyncratic productivities across working sectors lead to wage differences close to the data, as shown by the cross-sector comparison of mean earnings in

Figure 10: Participation rate, test taking behavior and informality



Notes: This figure shows, for each age, the fraction of the population participating the labor markets (top left), taking the public exam (top right), and working in the informal sector (bottom), in the calibrated model (red) and in the data (green).

Figure 11: Distribution of labor productivity by working sector



Notes: This figure shows, for each working sector, the fraction of agents at each point in the grid for idiosyncratic productivities. The grid in the x-axis is the estimated z-grid obtained from Table 3

Table 7. The average wage of formal workers is 3.3 times larger than informal workers, and the average wage in the public sector is 1.7 times the average wage of private (formal and informal) workers, not too far from the ratios observed in the Brazilian economy, 2.1 and 2.2, respectively.

Table 7: Mean effective earnings by working sector

	Formal/Informal	Public/Private
Model	3.3	1.7
Data	2.1	2.2

Notes: This table compares the ratio of mean effective (after tax) earnings across working sectors in the model and in the data. Sources: model simulations and 2013 PNAD.

We now turn to non-targeted outcomes related to the social security system. Table 8 shows how the model performs in replicating the social security deficit breakdown, the retirement modality choices, and the early retirement in the private sector observed in the data.

Table 8: Social security moments: model vs. data

	Model	Data
Public sector SS deficit (%GDP)	0.9%	1.1%
Overall SS deficit (% GDP)	2.0%	2.1%
Fraction of contrib. modality claims	31%	61%
Average age at retirement	58.0	54.8
Average years contributed at retirement	35.0	35.3

Notes: This table shows moments related to the social security system in the model and in the data. "Average age" and "Average years contributed" are for the contribution modality only. Sources: model simulations and Tafner et al. (2015).

The estimated overall social security deficit is 2.1% of GDP, matching the 2013 data - 2.0% of GDP - closely. Moreover, the model breakdown of social security deficit across public and private retirement sectors is also very close to what was observed in the data in that year, with the public (private) share slightly smaller (larger) than the official figures. The model also matches closely the retirement age and years of contribution at retirement. It misses, however, the fraction of retirement claims relative to the contribution modality.

Overall the model does a good job in reproducing the data in most relevant dimensions. We turn now to the core of our analysis, and evaluate the effects of social security reforms under demographic changes in the long run.

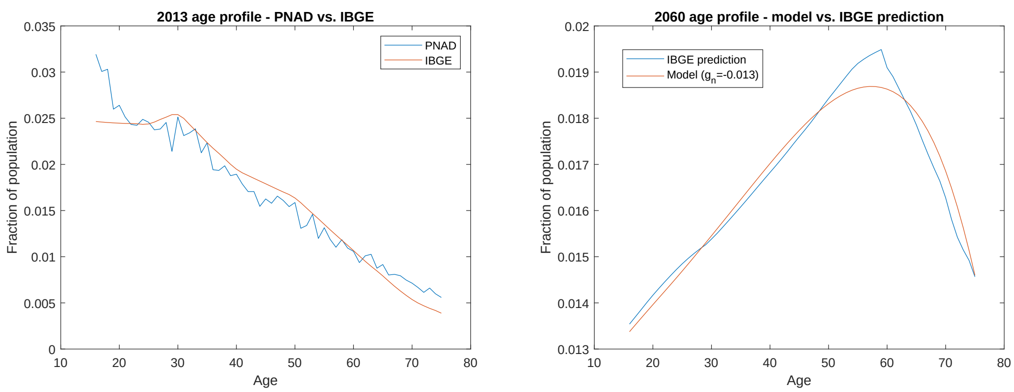
6 Social Security Reforms

What are the macroeconomic effects of reforming the social security system? In this section, we use the calibrated model to quantify the long run effects of two policies: the unification of the public and private retirement systems, and the imposition of a minimum retirement age. We describe how we compute the counterfactual demographics and detail the two policy reforms analyzed. We then discuss the quantitative results.

6.1 Counterfactual scenarios

Using data from the IBGE's Tables of Population Projections 2000-2060, we find the population growth rate that matches the age distribution of the Brazilian economy in 2060. This procedure yields a counterfactual growth rate of $g_n = -0.013$. Because the age distribution used in the model was obtained from the PNAD, we compare the age distribution in 2013 from these two sources. Figure 12 shows that the demographic structures are not far from each other (left panel), which gives us confidence in using these predictions for our counterfactual analysis, and shows that the new age profile from the model fits the Brazilian age profile in 2060 calculated by the IBGE (right panel).

Figure 12: 2013 and 2060 population age profiles, model vs. data



Notes: The left panel compares the age profile in the 2013 PNAD with the one in the IBGE's Tables of Population Projections 2000-2060. The right panel plots the model-implied population under the counterfactual growth rate of $g_n = -0.013$ versus the IBGE's projection for the male Brazilian population in 2060. Sources: 2013 PNAD and IBGE's Tables of Population Projections 2000-2060.

Under the new demographic structure, we compare the general equilibrium behavior of the model economy with and without social security reforms. The first reform analyzed is the unification of the social security schemes. In 2013, Law 12.618 imposed a ceiling on the social security benefits received by new entrants in the public sector. This cap is the same that limits private sector benefits, $y_{\max}$. The eligibility conditions did not change, neither did the benefits' formulas. The contribution to the social security system was limited to 11% of the minimum between the earnings of the public worker and $y_{\max}$. This reform approximated the two social security regimes, and intends to alleviate, at least in an *ex-ante* manner, the fiscal pressure of public retirement on public accounts. The second reform is at the core of most discussions regarding social security: an increase in the minimum retirement age. We choose to impose a minimum retirement age of 65, as it mimics what was implemented by the Brazilian government with the Constitutional Amendment 103 of November 12th, in 2019[31].

6.2 Results

We now quantify the macroeconomic impacts of social security reforms under population ageing. First, we evaluate the changes in aggregate variables such as the social security deficit as a fraction of GDP, the aggregate capital, the participation rate, and others. We then look at the changes in the life cycle behavior of agents implied by these social security reforms. Lastly, we perform a long-run welfare analysis.

Table 9 summarizes the effects of the demographic change and social security reforms on aggregate variables. Relative to the 2013 economy, population ageing (column "2060") increases significantly the social security deficit over GDP, the aggregate capital stock, and the capital-to-output ratio, shortens the participation in the labor markets, and lowers both informality and public sector applicants.

Table 9: The effects of social security reforms on equilibrium variables

	2013	2060	Unification	Minimum age	Both
	(levels)			(% change vs. 2060)	
SS deficit/GDP	2.0%	8.7%	-22.7	-31.7	-37.6
Public SS deficit/GDP	0.9%	2.8%	-78.7	-72.5	-90.7
Private SS deficit/GDP	1.1%	5.9%	+3.1	-12.9	-13.0
Consumption tax	20.7%	29.4%	-8.4	-13.5	-15.5
Avg. consumption (C)	R$ 17,246	R$ 18,981	+2.1	+0.7	+1.5
Avg. capital (K)	R$ 71,725	R$ 79,141	+3.9	+2.0	+3.7
Avg. output (Y)	R$ 25,946	R$ 27,749	+2.7	+0.6	+1.7
K/Y	2.58	2.83	+1.4	+1.1	+1.8
Participation rate	71.7%	61.7%	+1.1	-1.4	-0.7
Fraction of retirees	10.5%	27.0%	+3.2	-30.7	-30.8
Informality	19.1%	3.1%	-26.5	+29.5	+18.9
Public sector applicants	1.1%	0.8%	-20.3	-45.2	-32.9

Notes: This table shows the effects of different social security reforms on a selected group of aggregate variables. The "2013" column displays the variables in the 2013 economy, the "2060" column displays the variables in the 2060 economy without any reforms, and the remaining columns denote changes relative to the "2060" column.

The model predicts that participation rate will drop by ten percentage points, and that the share of retired people will go from 10.5% in 2013 to 27% in 2060. Moreover, agents that choose to stay at home account now for 38.3% of the total population. Consequently, overall social security deficit

[31] This is the age for men. For women, the minimum retirement age is 62 years old. We are also abstracting from a large number of transition rules.

jumps from 2% of GDP to 8.7%, with its public portion going from 0.9% to 2.7% of GDP, and that of the private sector rising from 1.1% to 5.9% of GDP. Those numbers tighten up the government's budget constraint, inducing an increase in the consumption tax rate from 20.7% to 29.4%.

Many countries around the world are experiencing a similar process of population ageing. Several of them have overly generous dual social security systems that typically favor public sector workers. The result above is an indication that these economies, and not only Brazil, will experience in the future a serious deterioration of their fiscal conditions if they do not change their retirement rules.

Despite the large increase of the consumption tax rate, average output and consumption are larger in the 2060 economy than in the calibrated economy. Why? In the counterfactual exercises we kept constant the age-efficiency profile, displayed in Figure 8. However, the 2060 demography has relatively more (less) people aged 35-55 (16-30) than the 2013 demography (Figure 12). Therefore, the productivity of the workers in the 2060 economy tend to be larger, increasing the aggregate labor efficiency in the economy and implying an increase in aggregate product (which is also driven by greater savings) and consumption. If the population ageing were even more intense, we would start to have more workers at the "other end" of the age-efficiency profile (over 65 years old - as shown in Figure 8). In this case, aggregate product and consumption would be lower than in the 2013 baseline.

The last three columns of Table 9 present the aggregate implications of the social security reform simulations compared to a 2060 scenario with no reforms. In all of them we kept the 2060 demographics and all other parameters unchanged. The third column presents the simulation results of the unification of social security systems, the fourth refers to the implementation of a minimum retirement age of 65, and the last column to the right presents figures for the case in which both reforms are implemented.

A key result of the reforms is the reduction of the social security deficit. When both reforms are jointly implemented social security deficit falls by almost 40% when compared to a scenario without any reform. One of the main reasons for this result is the sharp decrease in the share of retired people, 31%. Both reforms reduce social security deficit, but only the imposition of a higher minimum age reduces the number of retired people, given that now people stay longer in the labor markets. However, total social security deficit will remain relatively high as a proportion of GDP, around 5.4%.

We now analyze the break down of the deficit reduction. The deficit of the public sector in 2060, if nothing is done, is estimated to jump to 2.8% of GDP, a threefold increase. With the reforms it would be to only 0.3% of GDP. In contrast, we estimate that the reforms will reduce the private sector deficit in 2060 from 5.9% of GDP to 5.2%. This asymmetry happens because of general equilibrium effects. As the reforms make a public job less attractive, they lead to both a reduction of the “late-comers” in the public sector - i.e., those agents that work in the private sector until they are 40 years old, then try and enter the public sector to enjoy the good retirement conditions of the sector - and to an increase in the average productivity of the public worker - because the public sector is an absorbing state, a deterioration in the retirement conditions reduce incentives for low-productivity agents to seek a public career. Hence, low-productivity agents which, absent the reforms, would be in the public sector, now make a career in the private sector, contributing to high levels of private deficit. Hence, the impact of the unification is quite strong in reducing the deficit of the public sector, as expected, but increases slightly that of the private sector (second and third lines of “Unification” column).

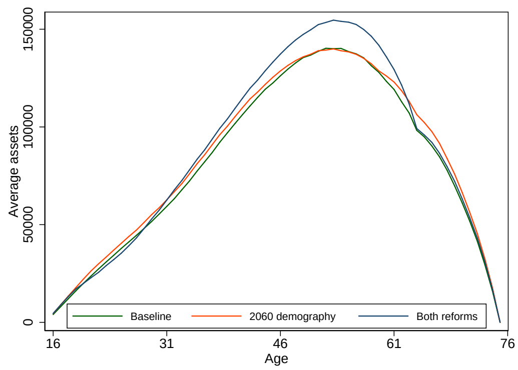
The column “Minimum age” of Table 9 compares the steady state equilibria without reforms and with the introduction of a minimum retirement age of 65 years old. In this case social security deficit drops by 31.7% with respect to the no reform case. Consequently, consumption taxation would be 13% smaller, leading to an increase in average consumption, output and capital. In both reforms the disincentive to enter the public sector increases, leading to a drop in the number of test takers. When examining their joint impact (the ”Both” column) the number of public sector applicants falls by a third, when comparing to the no reform scenario in 2060. Macroeconomic variables such as average output, capital and consumption also expand, the latter by 1.5%. That is good news, social security reform not only reduces the deficit as expected, but expands income of the population as a whole.

We now discuss the response of savings, both in aggregate and over the life cycle. Table 9 shows that average capital, i.e. savings, increases in 2060 with respect to 2013. As already discussed, the 2060 economy has a larger share of workers in the most productive part of their working careers, where they save up the most out of labor income. The reforms will lead to further expansion of savings. In the simulation with both reforms being implemented, aggregate savings increase by almost 4% with respect to the no reform scenario. In addition to the reduction of consumption taxation, agents also increase their savings to smooth consumption as the pension system becomes less generous.

The above rationale is clear in Figure 13. The figure displays average savings over the life cycle. An older population induces more saving at almost every age, as the 2060 line is nearly everywhere above that of 2013. However, for our purposes, the most interesting feature is the jump of the savings profile in a 2060 world with social security reforms. There is a significant increase in total average assets from around 30 years old to 65 years old. After this age, on average, individuals’ assets fall below the levels of the no reform world. People are saving more to compensate the fall of retirement income caused by the changes in social security.

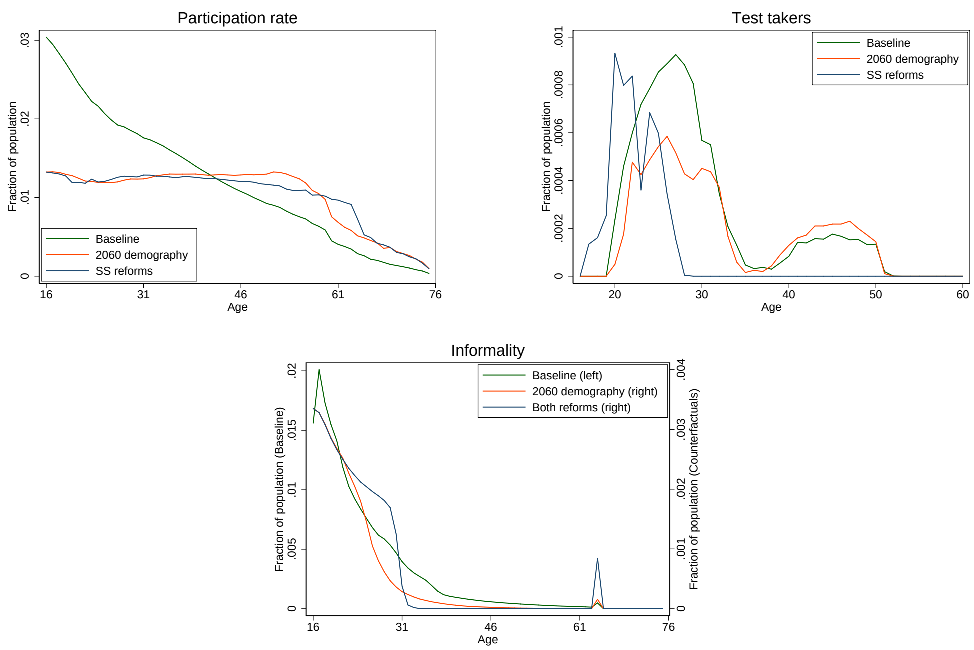
Figure 14 depicts the economy’s responses with respect to informality, labor market participation, and public sector application decisions. As also displayed in Table 9, informality falls significantly

Figure 13: The response of average savings over the life cycle



Notes: This figure plots asset accumulation over the life cycle in 2013 Brazilian Reais.

Figure 14: The responses of participation rate, test taking behavior and informality



Notes: This figure shows, for each age, the fraction of the population participating the labor markets (top left), taking the public exam (top right), and working in the informal sector (bottom), in the calibrated model (red) and in the data (green).

between 2013 and 2060. This is mostly due to population ageing, as in the 2013 simulated economy a large part of the informal workforce was composed of younger people. Social security reforms will not modify much this picture, and will lead to a modest increase in informality. Likewise, demographic changes seem to be the most important factor to explain future variations in the job market participation rate. The most relevant change caused by the reforms is a significant increase of the participation of people between 60 to 65 years older in the job market, due to the new minimum retirement age. Public sector application decisions change significantly in each scenario. From 2013 to 2060 there is a substantial reduction in the proportion of applicants, but the shape of the age profile did not change much. In both cases the distribution is twin peaked as older people try to enter the public sector to benefit from its more advantageous retirement system. The final effect of the social security reform is, first, to eliminate this application peak of older worker. There are no longer gains in migrating to the public sector near retirement. The second difference is that there is now a concentration of younger applicants, so that the model predicts even longer worker tenures in the public sector.

6.3 Welfare

We now analyze the long-run welfare consequences of the social security reforms. We measure welfare as the proportional change in consumption for all agents in the economy, Δ_w, that equalizes the average utility between the steady state equilibria. As in the previous section, we compare welfare changes between equilibria with different social security reforms relative to the equilibrium where the Brazilian population has grown older but the government did not adjust the social security rules.

To ease notation, let the prime superscripts denote equilibrium variables in the steady states where reforms are implemented, and variables without the prime superscripts denote the steady state without reforms. The change in welfare is given by:

$$\Delta_w \in \mathbb{R}: \quad \sum_s \mu(s) u(d^c(s), \bar{H} - d^h(s), Y_G) = \sum_s \mu'(s) u((1 - \Delta_w) d^{c\prime}(s), \bar{H} - d^{h\prime}(s), Y_G') \tag{4}$$

Table 10 reports welfare variation following the above definition. Positive values for Δ_w mean that agents are better off in the steady states where social security reform are implemented. The unification of the pension systems and the imposition of the minimum retirement age have similar impact on aggregate welfare, increasing it by 2% and 1.8%, respectively, in consumption equivalent terms relative to the no reforms scenario. When jointly implemented, there is a 2.3% aggregate welfare gain. Moreover, imposing both reforms at once results in positive welfare gains for all sectors in the economy, although in some isolated cases (e.g., government workers with the introduction of higher minimum retirement age) there are sizable steady state losses.

Table 10: Welfare gains in consumption equivalence

Reform:	Unification	Min age	Both
Overall	2.0%	1.8%	2.3%
Private sector	2.3%	7.7%	8.0%
Government	4.7%	-4.3%	5.3%
Retirees	-1.9%	8.4%	7.2%

Notes: Each column shows the long run welfare gains in consumption equivalence of implementing the social security reforms, relative to a steady state where no reforms are implemented (Δ_w in Equation (4)). At the sectoral level we normalize the measures, $\mu(s_m)$, by the size of the respective sector, $\sum_{s_m} \mu(s_m)$.

7 Conclusion

Population ageing is putting a very strong strain on the solvency of social security systems around the world. The problem only tends to worsen, as this movement will accentuate in the future, so that the ratio between retired population and active workers will increase steadily. Add to it excessively generous retirement conditions, particularly in what concerns the public sector, and the end result is financially unsustainable pension systems that will tend to absorb a growing share of tax revenues.

Brazil is no different, if anything it is an acute case of fiscal irresponsibility and can be used as an experiment for countries in similar situation. We modeled in detail the Brazilian economy and its social security system - not that far from many systems in the world - and simulated it plugging the demographic conditions projected to 2060. We found that, if nothing is changed, population ageing alone will increase social security deficit from around 2% today to more the 8% in less than 45 year from now. Given the necessary increase in taxation, the impact on the economy will be strong. Hence, the first lesson is that inertia and inaction will have a very high cost in the future.

We introduce in the economy different reforms implemented in the recent past or currently in consideration. These reforms are not far from those discussed in many other countries. We find that the unification of social security systems, in a world were public employees face very generous retirement conditions, is able to decrease considerably the social security deficit. Public workers stay longer in the job, increasing their savings, to compensate for the reduction in retirement benefits. Moreover, the reforms will have in the long run positive effects on aggregate savings and output, at the same time that it changes considerably the decision to join the public sector. Now, people apply earlier to a public job and, on average, they are more qualified.

When the minimum retirement age is increased and longer contributions are implemented, social security deficit is further reduced. Early retirement, hence, has a very high cost: a large fraction of workers retire today before they reach 60 years of age, and the reform forces them to work until 65 years of age. At the same time, simulations found that savings, consumption and output increase in the long run. The welfare, on average, also increases. Hence, the society as a whole has a lot to gain with sensible social security modifications.

Of course, given the political resistance of public workers against pension reforms - or of the whole population against any reform as it seems to be the case in France, for instance - the implementation of some of these changes will be very difficult if not impossible in the short run. That is so because many will lose during the transition to the new regime. However, the figures estimated in the present study show that the gains in the future are large and suggest it is worth facing the opposition of interest groups.

References

Almeida, R. and Carneiro, P. (2012). Enforcement of labor regulation and informality. *American Economic Journal: Applied Economics*, 4(3):64–89.

Attanasio, O. (1999). Consumption. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1, Part B, chapter 11, pages 741–812. Elsevier, 1 edition.

Bacchetta, M., Ernst, E., Bustamante, J. P., et al. (2009). *Globalization and Informal Jobs in Developing Countries*. International Labour Organization Geneva.

Beshears, J., Choi, J. J., Laibson, D., and Madrian, B. C. (2011). Behavioral economics perspectives on public sector pension plans. Working Paper 16728, National Bureau of Economic Research.

Blöndal, S. and Scarpetta, S. (1999). The retirement decision in oecd countries. OECD Economics Department Working Papers 202, OECD Publishing.

Chen, K. (2010). A life-cycle analysis of social security with housing. *Review of Economic Dynamics*, 13(3):597 – 615.

Conesa, J. C. and Krueger, D. (1999). Social Security Reform with Heterogeneous Agents. *Review of Economic Dynamics*, 2(4):757–795.

De Nardi, M., Fella, G., and Pardo, G. P. (2016). The implications of richer earnings dynamics for consumption and wealth. Working Paper 21917, National Bureau of Economic Research.

de Paula, u. and Scheinkman, J. A. (2011). The informal sector: An equilibrium model and some empirical evidence from brazil. *Review of Income and Wealth*, 57:S8–S26.

Dos Reis, A. C. and Zilberman, E. (2014). The insurance rrle of public employment. *Working Paper*.

dos Santos, M. and Cavalcanti, T. (2015). (Mis)Allocation Effects of an Overpaid Public Sector. 2015 Meeting Papers 1094, Society for Economic Dynamics.

dos Santos, M. and Pereira, T. N. (2010). Moving to a consumption-cased tax system: a quantitative assessment for brazil. *Revista Brasileira de Economia*, 64.

Fajardo, J., Ornelas, J. R. H., and Farias, A. R. d. (2012). Estimating risk aversion, risk-neutral and real-world densities using brazilian real currency options. *Economia Aplicada*, 16:567 – 577.

Ferreira, P. C. and do Nascimento, L. G. (2005). Welfare and growth effects of alternative fiscal rules for infrastructurei investment in brazil. *Manuscript.*

Ferreira, P. C. and dos Santos, M. R. (2013). The effect of social security, health, demography and technology on retirement. *Review of Economic Dynamics*, 16(2):350 – 370.

Ferreira, P. C. and Gomes, D. B. (2017). Health care reform or more affordable health care? *Journal of Economic Dynamics and Control*, 79:126 – 153.

Gandelman, N. and Hernandez-Murillo, R. (2015). Risk Aversion at the Country Level. *Review*, 97(1):53–66.

Glomm, G., Jung, J., and Tran, C. (2009). Macroeconomic implications of early retirement in the public sector: The case of brazil. *Journal of Economic Dynamics and Control*, 33(4):777 – 797.

Gomes, D., Santos, C., and Iachan, F. (2019). Labor Earnings Dynamics with a Large Informal Sector. 2019 Meeting Papers 793, Society for Economic Dynamics.

Gomes, V., Bugarin, M. N. S., and Ellery-Jr, R. (2005). Long-run Implications of the Brazilian Capital Stock and Income Estimates. *Brazilian Review of Econometrics*, 25(1).

Gruber, J. and Wise, D. A. (2002). Social security programs and retirement around the world: Micro estimation. Working Paper 9407, National Bureau of Economic Research.

Gustman, A. L. and Steinmeier, T. L. (2005). The social security early entitlement age in a structural model of retirement and wealth. *Journal of Public Economics*, 89(2):441 – 463.

Guvenen, F., Karahan, F., Ozkan, S., and Song, J. (2015). What do data on millions of u.s. workers reveal about life-cycle earnings risk? Working Paper 20913, National Bureau of Economic Research.

Hansen, G. D., Hsu, M., and Lee, J. (2014). Health insurance reform: The impact of a medicare buy-in. *Journal of Economic Dynamics and Control*, 45:315 – 329.

Holzmann, R. and Hinz, R. (2005). *Old-Age Income Support in the 21st Century.* The World Bank.

Huggett, M. (1996). Wealth distribution in life-cycle economies. *Journal of Monetary Economics*, 38(3):469–494.

Huggett, M. and Ventura, G. (1999). On the Distributional Effects of Social Security Reform. *Review of Economic Dynamics*, 2(3):498–531.

Immervoll, H., Levy, H., Nogueira, J. R., O'Donoghue, C., and Siqueira, R. B. d. (2006). The Impact of Brazil's Tax-Benefit System on Inequality and Poverty. IZA Discussion Papers 2114, Institute for the Study of Labor (IZA).

Imrohoroglu, S. and Kitao, S. (2012). Social security reforms: Benefit claiming, labor force participation, and long-run sustainability. *American Economic Journal: Macroeconomics*, 4(3):96–127.

Jung, J. and Tran, C. (2012). The extension of social security coverage in developing countries. *Journal of Development Economics*, 99(2):439 – 458.

Kings, J., Turkisch, E., and Manning, N. (2007). Public sector pensions and the challenge of an ageing public service. *OECD Working Papers on Public Governance*.

Meghir, C., Narita, R., and Robin, J.-M. (2015). Wages and informality in developing countries. *American Economic Review*, 105(4):1509–46.

Morandi, L. (2016). Novas estimativas do estoque de capital fixo brasileiro - 1950:2014. *Séries Históricas - FGV Instituto Brasileiro de Economia*.

OECD (2015). Pensions at a glance. *OECD Publishing*.

Parente, S. L. and Prescott, E. C. (2002). *Barriers to Riches*, volume 1 of *MIT Press Books*. The MIT Press.

Pereira, R. A. d. C. and Ferreira, P. C. (2010). Avaliacao dos impactos macro-economicos e de bem-estar da reforma tributaria no brasil. *Revista Brasileira de Economia*, 64:191 – 208.

Tafner, P., Botelho, C., and Erbisti, R. (2015). Reforma da previdência: a visita da velha senhora. *Editora Gestão Pública*.

Tauchen, G. (1986). Finite state markov chain approximations to univariate vector autoregressions. *Economic Letters*, 20(2):177–181.

Ulyssea, G. (2017). Firms, informality and development: Theory and evidence from brazil. *Working Paper*.

Vestad, O. L. (2013). Labour supply effects of early retirement provision. *Labour Economics*, 25:98 – 109. European Association of Labour Economists 24th Annual Conference, Bonn, Germany, 20-22 September 2012.

Appendix A Agents' Stationary Distribution

In this section, we formally derive the stationary distribution for the agents in the economy.

Given the assumptions on the entrants detailed in the main text, we have:

$$\mu_P(s_P) = \begin{cases} \varphi_1 \bar{\Gamma}(z) & \text{if} \quad s_P = (1, 0, z, 0, 0) \\ 0 & \text{for all other } s_P = (1, a, z, x, t_c) \end{cases}$$

and $\mu_m(s_m) = 0$ for all other sectors $m = \{R, G\}$ at $j = 1$, where $\bar{\Gamma}$ represents the stationary distribution for the Markov process $\{z\}$. We now detail the construction of the measures of agents in all sectors when $j > 1$.

The measure of each private agent in the economy, (j, a', z', x', t'_c), can be written as the sum of two terms. The first one considers the mass of all agents that were in the private sector and voluntarily remained there. The second term takes into account the private workers that applied to the public sector and failed to get in. Both the terms take into account the measure $\mu_P(j-1, a, z, x, t_c)$.

Within each term, we consider the transition probability of the idiosyncratic productivity, $\Pi(z, z')$, the optimal amount saved by the agents, $d^a(s)$, the updated average past earnings and updated time of contribution, conditional on optimal labor and informality choices, $\tilde{x}'$ and $\tilde{t}'_c$, retirement decisions $d^{ss}(s)$, the public sector application decision, $d^{ap}(s)$ and the probability of succeeding in the public sector exam.

The distribution for the formal private sector workers across ages $j = \{2, ..., J\}$ is recursively given by:

$$\mu_P(j, a', z', x', t'_c) = \frac{\psi_{j-1}}{1+g_n} \cdot \Bigg\{ \sum_{s_P|j-1} \Pi(z, z') \mathbb{I}_{\{d^a(s_P)=a'\}} \mathbb{I}_{\{\tilde{x}'=x'\}} \mathbb{I}_{\{\tilde{t}'_c=t'_c\}} (1 - d^{app}(s_P))(1 - d^{ss}(s_P)) \mu_P(s_P) + \sum_{s_P|j-1} \Pi(z, z') \mathbb{I}_{\{d^a(s_P)=a'\}} \mathbb{I}_{\{\tilde{x}'=x'\}} \mathbb{I}_{\{\tilde{t}'_C=t'_C\}} d^{app}(s_P) \bar{q} \mu_P(s_P) \Bigg\}$$

where the updated average past earnings and length of contribution to the social security system can be written as a function of the optimal labor decision $d^h(s_P)$, informality decision $d^{inf}(s_P)$ and application to the public sector decision, $d^{app}(s_P)$:

$$(\tilde{x}', \tilde{t}'_C) = \begin{cases} \left(\frac{x(j-2) + min\{(1-c_{app}(j-1)d^{app}(s_P))w_f e^{z+\eta_{j-1}}, y_{\max}\}}{j-1}, \; t_c + 1 \right) & \text{if works formally} \\ (x, t_c) & \text{otherwise} \end{cases}$$

The distribution of the public servants has two components. The first one takes into account the private workers who took the test and succeeded. The second considers the decision of public workers who did not ask for retirement:

$$\mu_G(j, a', z', t'_C, t'_G) = \frac{\psi_{j-1}}{1+g_n} \cdot \Bigg\{ \sum_{s_P|j-1} \mathbb{I}_{\{z=z'\}} \mathbb{I}_{\{d^a(s_P)=a'\}} \mathbb{I}_{\{\tilde{x}'=x'\}} \mathbb{I}_{\{\tilde{t}'_c=t'_c\}} d^{app}(s_P)(1-\bar{q})\mu_P(s_P) + \sum_{s_G|j-1} \mathbb{I}_{\{z=z'\}} \mathbb{I}_{\{d^a(s_G)=a'\}} \mathbb{I}_{\{t_c+1=t'_c\}} \mathbb{I}_{\{t_G+1=t'_G\}} (1-d^{ss}(s_G))\mu_G(s_G) \Bigg\}$$

where $\tilde{x}'$ and $\tilde{t}'_c$ have the same definition as before.

The distribution of the retirees is also composed by two parts. First, we have agents who already were retired from the labor force. Second, we account for the ones who recently asked for retirement:

$$\mu_{RP}(j, a', z', b') = \frac{\psi_{j-1}}{1+g_n} \cdot \Bigg\{ \sum_{s_R|j-1} \Pi(z, z') \mathbb{I}_{\{d^a(s_R)=a'\}} \mathbb{I}_{\{b'=b\}} \mu_{RP}(s_R) + \sum_{s_P|j-1} \Pi(z, z') \mathbb{I}_{\{b(s_P)=b'\}} \mathbb{I}_{\{d^a(s_P)=a'\}} d^{ss}(s_P)\mu_P(s_P) \Bigg\}$$

and a similar equation applies to compute the distribution of the retirees in the public sector, which we omit for brevity.

Appendix B Defining the Stationary Competitive Equilibrium

In this Appendix, we define the recursive, stationary equilibrium of the model.

A *recursive competitive equilibrium* consists of value functions $V : S_m \to \mathbb{R}$ for all sectors $m \in \{P, G, R\}$, policy functions: (i) $d^h : S_m \to \{0, H\}$, for the optimal labor decision, asset holdings $d^a : S_m \to \mathbb{R}_+$ and consumption $d^c : S_m \to \mathbb{R}_{++}$ for all $m \in \{P, G, R\}$; (ii) $d^{ss} : S_m \to \{0, 1\}$ retirement decisions for $m \in \{P, G\}$; (iii) $d^{inf} : S_m \to \{0, 1\}$, for the optimal decision of working for the informal sector for $m \in \{P, R\}$; (ii) $d^{app} : S_P \to \{0, 1\}$ for the optimal decision of application to the public sector; competitive prices $\{r, w_f, w_i\}$, age dependent but time invariant measures of agents $\mu_m(s_m)$, government transfers ζ, taxes, an amount of public good, Y_G, and a threshold score $\bar{q}$ such that:

(1) The value and policy functions solve the problem of the agents;

(2) Formal private firms maximize profits given $\{r, w_f\}$ and informal wages are given by $w_i = A_i$;

(3) The individual and aggregate behavior are consistent:

$$K' = \sum_{m \in \{P,G,R\}} \sum_{s_m} d^a(s_m)\mu_m(s_m)$$

$$K = \frac{K'}{1+g_n}$$

$$N_f = \sum_{m \in \{P,R\}} \sum_{s_m} e^{z+\eta_j}(1-d^{inf}(s_m))d^h(s_m)\mu_m(s_m)$$

$$N_i = \sum_{m \in \{P,R\}} \sum_{s_m} e^{z+\eta_j} d^{inf}(s_m)d^h(s_m)\mu_m(s_m)$$

(4) The government chooses $\bar{q}$ in order to balance people coming in and out:

$$\bar{N}_G = \sum_{s_G} \mu_G(s_G)$$

(5) Public goods' consistency:

$$Y_G = \sum_{s_G} e^{z+\eta_j} \mu_G(s_G)$$

(6) Final good market clears:

$$\sum_{m \in \{P,G,R\}} \sum_{s_m} d^c(s_m)\mu_m(s_m) + K' + C_g = Y_f + Y_i + (1-\delta)K$$

(7) τ_c balances the government budget constraint:

$$\sum_{m,\, s_m} \tau_c \cdot d^c(s_m)\mu_m(s_m) + \sum_{m,\, s_m} \tau_k r \cdot a\mu_m(s_m) + \tau_y(P)w_f N_f + \tau_{ss}(P)\sum_{s_P}(1-d^{inf}(s_P))\min\{w_f e^{z+\eta_j} d^h(s_P), y_{\max}\}\mu_P(s_P) =$$
$$\sum_{s_G}(1-\tau_y(G)-\tau_{ss}(G))(1+\theta)w_f e^{z_G+\eta_j} H \cdot \mu_G(s_G) + \sum_{s_R}(1-\tau_b)b \cdot \mu_R(s_R) + C_g$$

(8) Bequests are rebated to the living ones:

$$\zeta = \frac{1+r}{1+g_n} \sum_{m,\, s_m} d^a(s_m)(1-\psi_j)\mu_m(s_m)$$

Appendix C Computing the Stationary Competitive Equilibrium

In this appendix we detail the computational methods used to quantitatively assess the macroeconomic consequences of social security reforms.

We numerically solved the model in Fortran 90. In order to do so, we discretized the asset space, the average past earnings space, the income process space and the social security benefits space. We did so in 52, 10, 5 and 42 points, respectively.

The grid on assets goes from R\$0 to R\$3,790,202, with its points concentrated over the lower bound[32]. The grid for x is equally spaced between 0 and $y_{\max}$. The grid for b linearly spaced between[33] R\$0 and R\$100,000.

The algorithm to find the general equilibrium used was an adaptation from the algorithm that is commonly used in the literature[34], including a fixed point over $\bar{q}$ to match $\bar{N}_G$. The steps used to compute the stationary equilibrium are:

1. Guess initial values for $\Theta \equiv (r, Y_G, \zeta, \tau_c, \bar{q})$;
2. Use the formal firm first order conditions to obtain w_f;
3. Solve the agents' problems backwards and find the respective policy functions;
4. Use the policy functions to compute the associated stationary distribution of households by forward induction;
5. Aggregate the individual decisions and find $\tilde{\bar{q}}$ such that $\bar{N}_G$ of the population is working as public servants;
6. Use individual decisions to calculate the implicit remaining variables $(\tilde{r}, \tilde{Y}_G, \tilde{\zeta}, \tilde{\tau_c})$;
7. Check whether $||\tilde{\Theta} - \Theta|| < \epsilon$. If not, update Θ, return to item 2 and iterate until convergence.

[32]We chose the upper bound as the smallest value such that no agent *in any state* chooses optimally this upper bound. The concentration in the grid follows a cubic polynomial.

[33]We chose a value for the upper bound large enough such that more than 99% of the benefits distributed in equilibrium are lower than that value.

[34]See Chen (2010) for an example.